四频差动陀螺的第二类闭锁效应

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提　要　本文探讨了四频差动陀螺的左旋模与右旋模之间的耦合效应，得出基本公式。在同旋模间的耦合为主的特殊条件下得到近似解析解，定量地描绘每一单陀螺处行波状态，而差动拍频闭锁的第二类闭锁效应，与我们近年的实验数据相符合。文中并指出削弱此效应的有效措施。此外，还得出正的相对比例因子修正项，是单陀螺相应项的推广。

前　言

自从76年对激光陀螺作全面的理论分析[1]以来，国内四频差动陀螺已取得重大进展，我们已进入实用样机阶段，理论上亦有重要的补充和发展[2,3,4,5,6,7,8]，但是，到目前为止，我们还只限于分析近独立的差动陀螺，还沒有来得及定量分析左旋陀螺与右旋陀螺的模耦合效应。作者在[1]中曾从物理直观出发作了一些定性的分析及估计（见该文P.P. 135～138），并提出要进行定量计算，由于实验工作很忙，至今沒有完成任务。

大约在两年前，我们在实验中发现了每个单陀螺皆为行波，而它们的拍频差被闭锁为0的第二类闭锁效应，并对它作了较系统的观察研究，揭开了研究陀螺间模耦合的序幕。由于国内大部分同行对此尙缺乏认识，故今年三月以交流形式提出来时不能被接受。但事实终归是事实，总会被人们所认识。本文是从理论角度计算此效应，得出与实验符合的结果。

第二类闭锁效应是陀螺间耦合的一个特殊效应，并且恰好能作近似解析解。陀螺间耦合的其它效应太复杂，至今无法作近似解析解，并沒有能完成计算机的计算。可以预见到：必须考虑陀螺间的耦合效应才能是眞正的定量理论。像用[1]的近独立理论于实验数据，得出原子线宽 $\gamma_{ab}/2\pi\cong 50MHz$ 及差损 $|\gamma_1-\gamma_2|\cong 1\times 10^{-6}$[3]，它的有效性是令人怀疑的。由后者作出理论猜想（见[2]的文献[5]）更缺乏可靠基础，必须重新估价。

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§1. 基本理论公式

取 z 轴沿环路方向，对环形激光器中激光的电矢量 $\vec{E}(z,t)$，可求出满足下列方程式[9,10]：

$$C^2\cdot\frac{\partial^2\vec{E}}{\partial z^2}-\frac{\partial^2\vec{E}}{\partial t^2}-\frac{1}{\epsilon_0}\cdot\overleftrightarrow{\sigma}\cdot\frac{\partial\vec{E}}{\partial t}+2\left(\hat{z}\cdot[\vec{\Omega}\times\vec{r}]\right)\frac{\partial^2\vec{E}}{\partial z\partial t}=\frac{1}{\epsilon_0}\frac{\partial^2\vec{P}}{\partial t^2},\tag{1}$$

式中
$\vec{P}\equiv\vec{P}(z,t)$：电极化强度矢量，
$\overleftrightarrow{\sigma}\equiv\overleftrightarrow{\sigma}(z)$：描述环路损耗的等效电导率张量，
$\vec{\Omega}$：转动的角速度，
$\vec{r}\equiv x\hat{x}+y\hat{y}+z\hat{z}$，
$\hat{x},\hat{y},\hat{z}$：x,y,z 轴的单位矢量，
C：光在真空中的速度，
$\epsilon_0\mu_0=\frac{1}{C^2}$．

为计算的肯定起见，设四频差动陀螺模式的排列如图1。图中 ν 为频率，“R、L”为右旋、左旋，“±”为正旋、负旋，“顺、逆”为顺时钟、逆时钟，“1,2,3,4”为模的次序，差频 $\nu_2-\nu_1$ 和 $\nu_4-\nu_3$ 为：

$$\left.\begin{aligned}\nu_2-\nu_1&=\nu_H+\nu_{转}+其它,\\ \nu_4-\nu_3&=\nu_H-\nu_{转}+其它,\\ \nu_{转}&=\frac{4A}{L\lambda}\cdot\Omega,\end{aligned}\right\}\tag{2}$$

ν_H 为磁光偏频量。最后的结果与图1的排列无关。

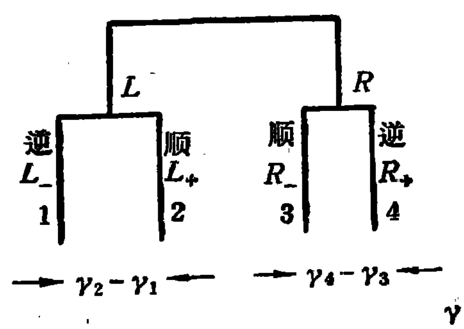

图 1

下面取 z 轴与模1同向，则波矢量 K_j 的正负见表1，偏振的单位矢量 $\hat{\varepsilon}_j$ 亦见表1，其中 $\hat{\varepsilon}_\pm$ 为正旋、负旋的单位矢量：*

$$\hat{\varepsilon}_\pm=\frac{1}{\sqrt{2}}(\hat{x}\pm i\hat{y}),\tag{3}$$

满足

$$\left.\begin{aligned}&\hat{\varepsilon}^*_\pm\cdot\hat{\varepsilon}_\pm=1,\\ &\hat{\varepsilon}^*_+\cdot\hat{\varepsilon}_-=\hat{\varepsilon}^*_-\cdot\hat{\varepsilon}_+=0.\end{aligned}\right\}\tag{4}$$

* 我们按理论物理的符号规则，它与光学的符号规定相反，后者为 $\hat{\varepsilon}_\pm=\frac{1}{\sqrt{2}}(\hat{x}\mp i\hat{y})$．

由于$\vec{P}(z,t)$的各分量近单色，故

$$\frac{\partial^2 \vec{P}(z,t)}{\partial t^2} \cong -\omega^2 \vec{P}(z,t), \tag{5}$$

又$\vec{E}(z,t)$可展成：

表 1

横次序 j	1	2	3	4
偏振	$L-$	$L+$	$R-$	$R+$
$\hat{\varepsilon}_j$	$\hat{\varepsilon}_-$	$\hat{\varepsilon}_+$	$\hat{\varepsilon}_-$	$\hat{\varepsilon}_+$
K_j	$\lvert K_1\rvert$	$-\lvert K_2\rvert$	$-\lvert K_3\rvert$	$\lvert K_4\rvert$

$$\left.\begin{aligned} &\vec{E}(z,t)=\sum_{j=1}^{4}E_j(t)e^{-i\Psi_j(t)+iK_jz}\hat{\varepsilon}_j+C.C., \\ &\Psi_j(t)=\omega_j t+\phi_j(t),\ |K_j|\cong K=\frac{\omega}{C}. \end{aligned}\right\} \tag{6}$$

把(5)、(6)式代入(1)式，用旋转波近似法，丢掉空间和时间的快速振盪项，可得到$E_j(t)$及$\Psi_j(t)$的运动方程式：

$$\dot{E}_1+\frac{\omega}{2}g_{11}E_1+\frac{\omega}{2}Im\left[i\sum_{j=2}^{4}g_{1j}E_je^{i(\Psi_1-\Psi_j)}\right]=-\frac{\omega}{2\epsilon_0}Im[P_1(t)], \tag{7-1}$$

$$(\dot{\phi}_1+\omega_1-\Omega_1)E_1+\frac{2K_1\vec{\Omega}\cdot\vec{A}}{L}E_1+\frac{\omega}{2}Re\left[i\sum_{j=2}^{4}g_{1j}E_je^{i(\Psi_1-\Psi_j)}\right]$$

$$=-\frac{\omega}{2\epsilon_0}\cdot Re[P_1(t)]. \tag{7-2}$$

同样有 $j=2,3,4$ 的方程。式中 Re、Im 代表实部和虚部，又

$$\left.\begin{aligned} &P_j(t)=\frac{1}{L}\int_0^L dz\,\hat{\varepsilon}_j^*\cdot\vec{P}(z,t)\,e^{i(\Psi_j-K_jz)}, \\ &g_{ij}=\frac{1}{\epsilon_0\omega}\cdot\frac{1}{L}\int_0^L dz\,\hat{\varepsilon}_i^*\cdot\overleftrightarrow{\sigma}\cdot\hat{\varepsilon}_je^{-i(K_i-K_j)z}, \\ &\frac{2\vec{A}\cdot\vec{\Omega}}{L}=\frac{\vec{\Omega}}{L}\cdot\int_0^L\vec{r}\times\vec{dz}=\frac{1}{L}\int_0^L dz(\hat{z}\cdot[\vec{\Omega}\times\vec{r}]). \end{aligned}\right\} \tag{8}$$

由于(σ_{ij})是个对称实矩阵，即

$$\sigma_{ij}^*=\sigma_{ij}=\sigma_{ji}, \tag{9}$$

故 $$g^*_{ij}=g_{ji},$$
或 $$(g_{ij})^+=(g_{ij}),\tag{10}$$
即(g_{ij})为厄米特矩阵。

举一个例子。假设损耗与左旋或右旋无关，但对 s 线偏振（沿 x 轴）与 p 偏振（沿 y 轴）有点不相等，$x-y$为主轴，即

$$\overset{\leftrightarrow}{\sigma}=\sigma_{xx}\hat{x}\hat{x}+\sigma_{yy}\hat{y}\hat{y},\tag{11}$$

则$\overset{\leftrightarrow}{\sigma}$可重新以基矢$\hat{\varepsilon}_\pm$作表象

$$\sigma_{\pm,\pm}\equiv\hat{\varepsilon}^*_\pm\cdot\overset{\leftrightarrow}{\sigma}\cdot\hat{\varepsilon}_\pm,\tag{12}$$

由此得矩阵

$$(\sigma_{+-})=\frac{1}{2}\begin{pmatrix}\sigma_{xx}+\sigma_{yy}, & \sigma_{xx}-\sigma_{yy}\\ \sigma_{xx}-\sigma_{yy}, & \sigma_{xx}+\sigma_{yy}\end{pmatrix}.\tag{13}$$

(8)式的各矩阵元素为：

$$\left.\begin{aligned}&\sigma_{11}=\sigma_{33}=\sigma_{13}=\sigma_{31}=\sigma_{--},\\&\sigma_{22}=\sigma_{44}=\sigma_{24}=\sigma_{42}=\sigma_{++},\\&\sigma_{12}=\sigma_{14}=\sigma_{32}=\sigma_{34}=\sigma_{-+},\\&\sigma_{21}=\sigma_{41}=\sigma_{23}=\sigma_{43}=\sigma_{+-},\\&\sigma_{ij}\equiv\hat{\varepsilon}^*_i\cdot\overset{\leftrightarrow}{\sigma}\cdot\hat{\varepsilon}_j.\end{aligned}\right\}\tag{14}$$

因为通常随偏振方向而变化的损耗百分比是不大的，故从(13)式有

$$|\sigma_{+-}|=|\sigma_{-+}|\ll\sigma_{++}=\sigma_{--};\tag{15}$$

又因为

$$\left.\begin{aligned}&(K_1-K_2)z\cong(K_1-K_3)z\cong\frac{2\omega}{C}z=\frac{4\pi z}{\lambda},\\&(K_1-K_4)z\cong\frac{1}{C}(\Omega_1-\Omega_4)z\cong\frac{2\pi}{C}\cdot\left(\frac{C}{2L}\right)z=\frac{\pi z}{L},\end{aligned}\right\}\tag{16}$$

故 g_{12}、g_{13}的大小严重地依赖于损耗的非均匀程度，g_{12}还直接依赖于与偏振方向有关的（$\sigma_{xx}\neq\sigma_{yy}$）的非均匀损耗，因而用圆偏振光工作的单陀螺的锁区较小，符合 [1] 的分析。另一方面，g_{14} 与带偏振方向的损耗的非均匀程度的关系就小得多。因此，一般应有

$$\left.\begin{aligned}&|g_{12}|\ll|g_{13}|\quad(\text{由于 15 式}),\\&|g_{12}|\ll|g_{14}|\quad(\text{由于 16 式})。\end{aligned}\right\}\tag{17}$$

下面从密度矩阵ρ推 $\rho(z,t)$ 及 $\rho(t)$．发光的Ne原子能级见图 2，有[10]

$$\left.\begin{aligned}&\omega'_a=\omega_a+a'g_a\mu_B B/\hbar,\\&\omega'_b=\omega_b+b'g_b\mu_B B/\hbar,\end{aligned}\right\}\tag{18}$$

$$\left.\begin{aligned}&g_a=1.295,\quad g_b=1.301,\\&\text{取}\qquad g_a\cong g_b=1.301,\end{aligned}\right\}\tag{19}$$

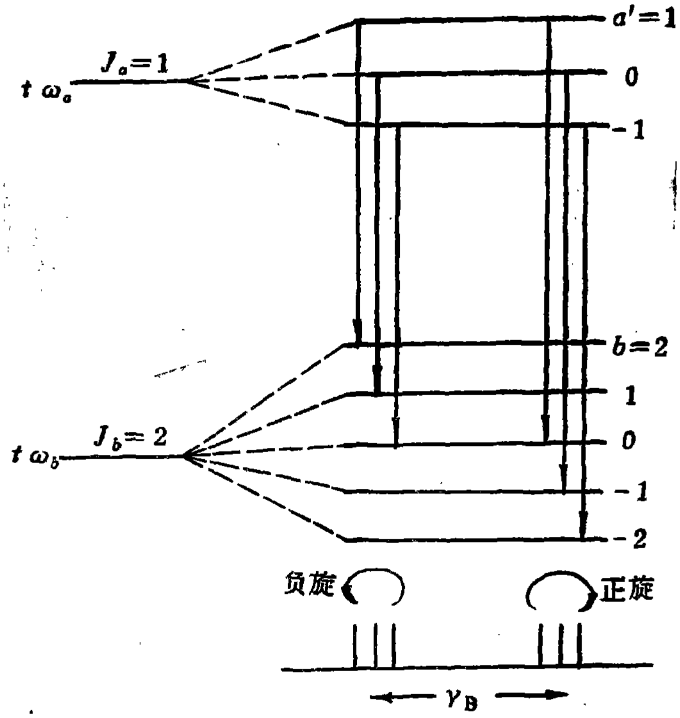

图 2**

$$\left.\begin{aligned} \nu_B &= \frac{2g_b\mu_B B}{h} = 3.54B,\ MHz, \\ \mu_B &= \frac{e\hbar}{2mC}, \end{aligned}\right\} \tag{20}$$

磁場B的单位用高斯。矩阵素

$$V_{a'b'} \equiv -\langle n_a J_a a' | e\vec{r}\cdot\vec{E} | n_b J_b b' \rangle, \tag{21}$$

$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z} = \frac{r\sin\vartheta}{2}(\hat{\varepsilon}_+^* e^{i\phi} + \hat{\varepsilon}_-^* e^{-i\phi}) + r\cos\theta\hat{z}, \tag{22}$$

故

$$\left.\begin{aligned} V_{a'b'} &= -(e\vec{r})_{a'b'}\cdot\vec{E}, \\ (e\vec{r})_{a'b'} &= p_{a'b'}[\hat{\varepsilon}_+^*\delta_{a',b'+1} + \hat{\varepsilon}_-^*\delta_{a',b'-1} + \hat{z}\delta_{a',b'}], \\ p_{a'b'} &= \begin{cases} \mp\frac{\sqrt{2}}{2}p[(J_b\pm a')(J_b\pm a'+1)]^{\frac{1}{2}}, & \text{当}a'=b'\pm1; \\ p[J_b^2-a'^2]^{\frac{1}{2}}, & \text{当}a'=b'; \end{cases} \\ p &\equiv \langle n_a J_a | er | n_b J_b \rangle. \end{aligned}\right\} \tag{23}$$

(23)式的 $p_{a'b'}$为[10]中$J_a=J_b-1$的特例，且$a'=b'\pm1$时比该处多乘$\sqrt{2}$*.

* 由于$e\vec{r}$用基矢$\hat{\varepsilon}_\pm^*$展开而不是用($\hat{x}\pm i\hat{y}$)展开，故差$\sqrt{2}$.

** 图2的$t\omega_a$、$t\omega_b$应为$\hbar\omega_a$、$\hbar\omega_b$，b应为b'.

对 n 个能态的密度矩阵 ρ 的运动方程为[10]

$$\left.\begin{aligned} & i\hbar\frac{\partial\rho}{\partial t}=[H,\rho]-\frac{i\hbar}{2}\{\Gamma.\rho\}, \\ & [H,\rho]\equiv H\rho-\rho H,\{\Gamma,\rho\}\equiv\Gamma\rho+\rho\Gamma, \\ & \Gamma=\begin{pmatrix}\gamma_1 & & 0\\ & \gamma_2 & \\ 0 & & \ddots \\ & & & \gamma_n\end{pmatrix}. \end{aligned}\right\} \tag{24}$$

式中 γ_j 为第 j 能态的自发衰变系数，H为能量矩阵,$H=H_0+V$, V为微扰能量矩阵。把(24)式的密度矩阵

$$\rho=\sum_{\Psi}\rho_{\Psi}|\Psi><\Psi| \tag{25}$$

转换成含$\vec{r}$、$\vec{v}$、t变量的密度矩阵 $\rho(\vec{r},\vec{v},t)$，有

$$\dot{\rho}\equiv\frac{\partial\rho(\vec{r},\vec{v},t)}{\partial t}+\vec{v}\cdot\nabla\rho(\vec{r},\vec{v},t)=-\frac{i}{\hbar}[H,\rho(\vec{r},\vec{v},t)]-\frac{1}{2}\{\Gamma,\rho(\vec{r},\vec{v},t)\}+\lambda, \tag{26}$$

其中

$$\lambda\equiv\begin{pmatrix}\lambda_1(\vec{r},\vec{v},t) & & 0\\ & \lambda_2(\vec{r},\vec{v},t) & \\ & & \ddots \\ 0 & & & \lambda_n(\vec{r},\vec{v},t)\end{pmatrix}, \tag{27}$$

$\lambda_j(\vec{r},\vec{v},t)\,d^3xd^3vdt$ 为激发到 j 能态和 d^3xd^3vdt 范围的原子数。特例：$\vec{r}$只含z,$\vec{v}$只含v_z写成 v，则(26)式可写成为：

$$\left.\begin{aligned} & \dot{\rho}\equiv\frac{\partial\rho}{\partial t}+v\frac{\partial\rho}{\partial z}=-\frac{i}{\hbar}[H,\rho]-\frac{1}{2}\{\Gamma,\rho\}+\lambda, \\ & \rho\equiv\rho(z,v,t). \end{aligned}\right\} \tag{26'}$$

用 $(a,a',a''\cdots)$、$(b,b',b''\cdots)$ 来代表图 2 的上、下能级各能态，则 (26′) 可改写成：

$$\left.\begin{aligned} & \dot{\rho}_{aa'}=-(\gamma_a+i\omega_{aa'})\rho_{aa'}-\frac{i}{\hbar}\sum_{c'=a'=-1}^{1}\sum_{c'=b'=-2}^{2} \\ & \qquad (V_{ac'}\rho_{c'a'}-\rho_{ac'}V_{c'a'})+\delta_{aa'}\lambda_a, \\ & \dot{\rho}_{bb'}=-(\gamma_b+i\omega_{bb'})\rho_{bb'}-\frac{i}{\hbar}\sum_{c'=a'}\sum_{c'=b'} \\ & \qquad (V_{bc'}\rho_{c'b'}-\rho_{bc'}V_{c'b'})+\delta_{bb'}\lambda_b, \\ & \dot{\rho}_{ab}=-(\gamma+i\omega_{ab})\rho_{ab}-\frac{i}{\hbar}\sum_{c'=a'}\sum_{c'=b'} \\ & \qquad (V_{ac'}\rho_{c'b}-\rho_{ac'}V_{c'b}), \\ & \rho_{ba}=\rho^*_{ab}. \end{aligned}\right\} \tag{28}$$

其中设

$$\left.\begin{aligned}&\gamma_{a'}=\gamma_{a''}=\cdots=\gamma_a, \quad \gamma_{b'}=\gamma_{b''}=\cdots=\gamma_b,\\&\gamma\equiv\gamma_{ab}=\frac{1}{2}(\gamma_a+\gamma_b)+\text{分子软硬碰撞项},\end{aligned}\right\}\tag{29}$$

后一式是常用的推广式。

在求出 $\rho(z,v,t)$ 后，可得 $\vec{P}(z,t)$ 及 $P_j(t)$：

$$\left.\begin{aligned}&\vec{P}(z,t)=\int_{-\infty}^{\infty}dvTr\,(\rho e\vec{r}).\\&P_j(t)=\frac{1}{L}\int_0^L dz\,\hat{\varepsilon}_j^*\cdot\vec{P}(z,t)e^{i\Psi_j(t)-iK_jz},\end{aligned}\right\}\tag{30}$$

$P_j(t)$ 就是 (7-1)、(7-2)式所需的“源头项”。

§2. 准确到三级微扰的公式

与通常一样，把(28)式先变成积分形式，然后以 V 的方次作为微扰的级别，用迭代法求得各级别的 ρ 的矩阵素：

$$\left.\begin{aligned}&\rho^{(0)}_{aa'}=\delta_{aa'}\frac{\lambda_a}{\gamma_a}, \quad \rho^{(0)}_{bb'}=\delta_{bb'}\frac{\lambda_b}{\gamma_b}, \quad \rho^{(0)}_{ab}=0,\\&\rho^{(1)}_{ab}=\cdots, \quad \rho^{(1)}_{aa'}=\rho^{(1)}_{bb'}=0,\\&\rho^{(2)}_{aa'}=\cdots, \quad \rho^{(2)}_{bb'}=\cdots, \quad \rho^{(2)}_{ab}=0,\\&\rho^{(3)}_{ab}=\cdots, \quad \rho^{(3)}_{aa'}=\rho^{(3)}_{bb'}=0.\end{aligned}\right\}\tag{31}$$

代入(30)式，求得各级别的 $P_j(t)$：

$$P_j^{(1)}(t)=-\left[\frac{10p^2N(t)}{\hbar^2Ku}\cdot Z(\xi_j',\eta)\right]E_j;\tag{32}$$

$P_j^{(3)}(t)$ 的烧孔效应项

$$\begin{aligned}=&\frac{iN(t)p^4}{\eta(Ku)^3\hbar^4}\Big\{\cdot 46\Big(\frac{1}{\eta_a}+\frac{1}{\eta_b}\Big)\Big[E_1^3(Z_i(\xi_1')-\eta Z_r'(\xi_1'))\\&+E_1E_3^2\mathcal{L}(\xi_{13}')\Big(\frac{Z_i(\xi_3')+Z_i(\xi_1')}{2}-\frac{\eta}{\xi_{13}'}\cdot\frac{Z_r(\xi_3')+Z_r(\xi_1')}{2}\Big)\Big]\\&+\Big(\frac{21}{\eta_a}+\frac{1}{\eta_b}\Big)\Big[E_1E_2^2\mathcal{L}(\xi_{12}')\Big(\frac{Z_i(\xi_2')+Z_i(\xi_1')}{2}-\frac{\eta}{\xi_{12}'}\cdot\frac{Z_r(\xi_2')+Z_r(\xi_1')}{2}\Big)\\&+E_1E_4^2\mathcal{L}\Big(\frac{\xi_1'-\xi_4'}{2}\Big)\Big(\frac{Z_i(\xi_1')+Z_i(\xi_4')}{2}-\eta\cdot\frac{Z_r(\xi_1')-Z_r(\xi_4')}{\xi_1'-\xi_4'}\Big)\Big]\Big\}\\&-\frac{N(t)p^4}{\eta(Ku)^3\hbar^4}\Big\{46\Big(\frac{1}{\eta_a}+\frac{1}{\eta_b}\Big)\Big[E_1^3(-\eta Z_i'(\xi_1'))-E_1E_3^2\cdot\eta\cdot\frac{Z_i(\xi_1')-Z_i(\xi_3')}{2\xi_{13}'}\\&+E_1E_3^2\Big(\frac{Z_i(\xi_3')+Z_i(\xi_1')}{2}-\frac{\eta}{\xi_{13}'}\cdot\frac{Z_r(\xi_3')+Z_r(\xi_1')}{2}\Big)\frac{\xi_{13}'\mathcal{L}(\xi_{13}')}{\eta}\Big]\end{aligned}$$

$$+\left(\frac{21}{\eta_a}+\frac{1}{\eta_b}\right)\left[E_1E_2^2\left(\frac{Z_i(\xi_2')+Z_i(\xi_1')}{2}-\frac{\eta}{\xi_{12}'}\cdot\frac{Z_r(\xi_2)+Z_r(\xi_1')}{2}\right)\frac{\xi_{12}'\mathcal{L}(\xi_{12}')}{\eta}\right.$$

$$-E_1E_2^2\cdot\eta\cdot\frac{Z_i(\xi_1')-Z_i(\xi_2')}{2\xi_{12}'}+E_1E_4^2\left(\frac{Z_i(\xi_4')+Z_i(\xi_1')}{2}-\eta\cdot\frac{Z_r(\xi_1')-Z_r(\xi_4')}{\xi_1'-\xi_4'}\right)$$

$$\left.\cdot\frac{(\xi_1'-\xi_4')\mathcal{L}\left(\frac{\xi_1'-\xi_4'}{2}\right)}{2\eta}-E_1E_4^2\cdot\eta\cdot\frac{Z_i(\xi_1')-Z_i(\xi_4')}{\xi'-\xi_4'}\right]\Bigg\},\qquad(33)$$

式中

$$\left.\begin{aligned}&\xi_j=\frac{\omega_j-\omega_0}{Ku},\ \xi_j'=\frac{\omega_j'-\omega_0}{Ku},\quad \eta=\frac{\gamma_{ab}}{Ku},\quad \eta_a=\frac{\gamma_a}{Ku},\\&\eta_b=\frac{\gamma_b}{Ku},\quad \xi_{12}'=\frac{\xi_1'+\xi_2'}{2},\quad \xi_{13}'=\frac{\xi_1'+\xi_3'}{2},\quad \omega_j'=\omega_j-\frac{(-1)^j\omega_B}{2},\\&Z(\xi_j)\equiv Z(\xi_j,\eta)=Z_r(\xi_j,\eta)+iZ_i(\xi_j,\eta)\text{：等离子体色散函数，}\\&\mathcal{L}(\xi)=\frac{\eta^2}{\xi^2+\eta^2},\ N(t)=\rho_{aa}^{(0)}-\rho_{bb}^{(0)}\text{对 } z \text{ 的平均值，}\end{aligned}\right\}\qquad(34)$$

并假设

$$\rho_{aa}^{(0)}=\rho_{a'a'}^{(0)}=\rho_{a''a''}^{(0)},\qquad \rho_{bb}^{(0)}=\rho_{b'b'}^{(0)}=\rho_{b''b''}^{(0)}=\cdots,\qquad(35)$$

又 ω_B 见(20)式，即

$$\frac{\omega_B}{2\pi}=\nu_B=\frac{2g_b\mu_BB}{h}=3.54B\quad MHz,\qquad(20)$$

ω_0 为静止原子发光的中心圆频率；

$\rho_1^{(3)}(t)$ 的粒子数脉动项

$=\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots.\qquad(36)$

(36)式的项实在太复杂，虽已计算出来，但本文对它不作分析，先忽略不计，故略去。$\rho_j^{(3)}(t)$ 亦类似 $\rho_1^{(3)}(t)$，只要相应的指标轮换。

把 $\rho_j^{(1)}(t)$ 和 $\rho_j^{(3)}(t)$ 代入(7-1)、(7-2)式，经过化简，可得

$$\left.\begin{aligned}&\frac{L}{C}\dot I_1=[\alpha_1-\beta_1I_1-\vartheta_{12}I_2-\vartheta_{13}I_3-\vartheta_{14}I_4]I_1\\&-\frac{\omega L}{C}Im\left[i\sum_{j=2}^4g_{1j}\sqrt{I_1I_j}e^{i(\Psi_1-\Psi_j)}\right],\\&\left[\dot\phi_1+\omega_1-\left(\Omega_1-\frac{2K_1\vec\Omega\cdot\vec A}{L}\right)\right]I_1\\&\qquad=[\sigma_1+\rho_1I_1+\tau_{12}I_2+\tau_{13}I_3+\tau_{14}I_4]I_1\\&-\frac{\omega}{2}Re\left[i\sum_{j=2}^4g_{1j}\sqrt{I_1I_j}e^{i(\Psi_1-\Psi_j)}\right],\end{aligned}\right\}\qquad(37)$$

式中

$$
\left.\begin{aligned}
I_j &= \frac{46(pE_j)^2}{10(Ku)^2\hbar^2}\cdot\frac{1}{\eta}\cdot\left(\frac{1}{\eta_a}+\frac{1}{\eta_b}\right),\\
G &= \frac{L}{C}\cdot\frac{10\omega p^2 N Z_i(0)}{\epsilon_0 K u\hbar},
\end{aligned}\right\} \tag{38}
$$

$$
\left.\begin{aligned}
&\alpha_j = g_j-\gamma_j,\quad g_j=\frac{GZ_i(\xi_j')}{Z_i(0)},\quad \gamma_j=\frac{L}{C}\cdot\frac{\omega}{Q_j}=\frac{L}{C}\cdot\omega g_{jj},\\
&\beta_j=\frac{G}{Z_i(0)}\cdot[Z_i(\xi_j')-\eta Z_r(\xi_j')],\\
&\vartheta_{13}=\frac{G}{Z_i(0)}\cdot\left[\frac{Z_i(\xi_3')+Z_i(\xi_1')}{2}-\frac{\eta}{\xi_{13}'}\cdot\frac{Z_r(\xi_3')+Z_r(\xi_1')}{2}\right]\mathcal{L}(\xi_{13}'),\\
&\vartheta_{12}=(\vartheta_{13})_{3\to2}\left(\frac{21}{\eta_a}+\frac{1}{\eta_b}\right)\bigg/\left[46\left(\frac{1}{\eta_a}+\frac{1}{\eta_b}\right)\right],\\
&\vartheta_{14}=\left\{\left(\frac{21}{\eta_a}+\frac{1}{\eta_b}\right)\bigg/\left[46\left(\frac{1}{\eta_a}+\frac{1}{\eta_b}\right)\right]\right\}\\
&\quad\cdot\frac{G}{Z_i(0)}\cdot\left[\frac{Z_i(\xi_4')+Z_i(\xi_1')}{2}-\eta\cdot\frac{Z_r(\xi_4')-Z_r(\xi_1')}{\xi_4'-\xi_1'}\right]\mathcal{L}\left(\frac{\xi_4'-\xi_1'}{2}\right),
\end{aligned}\right\} \tag{39}
$$

$$
\left.\begin{aligned}
&\sigma_j=\frac{C}{2L}\cdot\frac{G}{Z_i(0)}\cdot Z_r(\xi_j'),\\
&\rho_j=\frac{C}{2L}\cdot\frac{G}{Z_i(0)}\cdot[-\eta Z_i'(\xi_j')],\\
&\tau_{13}=\frac{C}{2L}\cdot\frac{G}{Z_i(0)}\cdot\left\{\left[\frac{Z_i(\xi_3')+Z_i(\xi_1')}{2}\right.\right.\\
&\qquad\left.\left.-\frac{\eta}{\xi_{13}'}\cdot\frac{Z_r(\xi_3')+Z_r(\xi_1')}{2}\right]\frac{\mathcal{L}(\xi_{13}')\xi_{13}'}{\eta}-\frac{\eta[Z_i(\xi_1')-Z_i(\xi_3')]}{2\xi_{13}'}\right\},\\
&\tau_{12}=(\tau_{13})_{3\to2}\cdot\left(\frac{21}{\eta_a}+\frac{1}{\eta_b}\right)\bigg/\left[46\left(\frac{1}{\eta_a}+\frac{1}{\eta_b}\right)\right],\\
&\tau_{14}=\left\{\left(\frac{21}{\eta_a}+\frac{1}{\eta_b}\right)\bigg/\left[46\left(\frac{1}{\eta_a}+\frac{1}{\eta_b}\right)\right]\right\}\cdot\frac{C}{2L}\cdot\frac{G}{Z_i(0)}\left\{\left[\frac{Z_i(\xi_1')+Z_i(\xi_4')}{2}\right.\right.\\
&\qquad\left.\left.-\eta\cdot\frac{Z_r(\xi_1')-Z_r(\xi_4')}{\xi_1'-\xi_4'}\right]\cdot\frac{\mathcal{L}\left(\frac{\xi_1'-\xi_4'}{2}\right)\cdot(\xi_1'-\xi_4')}{2\eta}-\frac{\eta[Z_i(\xi_1')-Z_i(\xi_4')]}{\xi_1'-\xi_4'}\right\}.
\end{aligned}\right\} \tag{40}
$$

应说明：上面沒有把 $P_j^{(3)}(t)$ 的粒子数脈动项考虑进去，否则 ϑ_{ij}、τ_{ij} 各函数要复杂得多。粒子数脈动项贡献一个不大的修正，在本文中略去，将来在其它讨论中可能出现。

(39)、(40)式有一重要结果：负旋模与正旋模间的相互作用（如：(1,2)、(1,4)）要比负旋与负旋间（如：(1,3)）或正旋与正旋间的相互作用（如 (2,4))弱，多乘一个小于 1 的因子：

$$a=\left(\frac{21}{\eta_a}+\frac{1}{\eta_b}\right)\Bigg/\left[46\left(\frac{1}{\eta_a}+\frac{1}{\eta_b}\right)\right]. \tag{41}$$

从图2可看出：这因子主要来源是前者几乎不竞争下能级的原因，正如[1]直观分析那样。

严格说来，我们的推导还缺乏考虑环路中折射率的突变效应，这在[9]中已作了，其结果是自然地得出非均匀散射效应。由于这样作将带来长篇推导，并且一維的非均匀散射仍与环路中二維散射有区别，故本文不去作，只接受其物理结果，补进方程式中去。我们按照[2]的作法，把非均匀散射与损耗的综合效果用一些复合的复数振幅反射系数来表示：

$$r_{ij}e^{i\delta_{ij}}; \tag{42}$$

为方便起见，用符号

$$\vartheta_{jj}\equiv\beta_j, \qquad \tau_{jj}\equiv\rho_j; \tag{43}$$

这样，方程组(37)可推广成：

$$\left.\begin{aligned}\frac{L}{C}\dot{I}_j&=\left(\alpha_j-\sum_{k=1}^{4}\vartheta_{jk}I_k\right)I_j+\sum_{k(\neq j)}{}'\,2r_{jk}\sqrt{I_jI_k}\sin(\Psi_k-\Psi_j-\delta_{jk}),\\ \dot{\Psi}_j&=\Omega_j+\sigma_j+\sum_{k=1}^{4}\tau_{jk}I_k-\frac{C}{L}\sum_{k(\neq j)}{}'\,r_{jk}\sqrt{I_k/I_j}\cos(\Psi_k-\Psi_j-\delta_{jk}),\end{aligned}\right\} \tag{44}$$

((37)式中 $2K_j\vec{\Omega}\cdot\vec{A}/L$ 已被吸收到 Ω_j 中去了。).

根据(17)式，可以预期，一般将有

$$\left.\begin{aligned}&r_{12}\cong r_{21}\ll r_{13}\cong r_{31},\\ &r_{12}\cong r_{21}\ll r_{14}\cong r_{41}\text{等},\end{aligned}\right\} \tag{45}$$

另一方面，由于

$$\left.\begin{aligned}&|\dot{\Psi}_1-\dot{\Psi}_2|/2\pi\approx0.3\sim1MHz,\text{等},\\ &|\dot{\Psi}_1-\dot{\Psi}_3|/2\pi\approx|\dot{\Psi}_1-\dot{\Psi}_4|/2\pi\approx200MHz.\end{aligned}\right\} \tag{46}$$

故(1,3)间、(1,4)间的耦合又被高拍频阶削弱。因此，综合看来，是(1,2)间耦合强或(1,3)、(1,4)间强得看实验条件。本文只讨论(1,2)和(3,4)之间占压倒强度的特例，因此，只保留，

$$r_{12},\ r_{21},\ r_{34},\ r_{43}\ \text{项}. \tag{47}$$

§3. 第二类闭锁效应

作一点变换。令

$$\left.\begin{aligned}&I_{21}\equiv\frac{I_2+I_1}{2}, \qquad i_{21}\equiv\frac{I_2-I_1}{2},\\ &I_{43}\equiv\frac{I_4+I_3}{2}, \qquad i_{43}\equiv\frac{I_4-I_3}{2},\end{aligned}\right\} \tag{48}$$

$$\left.\begin{aligned}&\Psi_2-\Psi_1=\phi_{21}+\pi+\frac{\delta_{12}-\delta_{21}}{2},\\&\frac{\delta_{12}+\delta_{21}}{2}=\frac{\pi}{2}-\varepsilon_{12},\\&\Psi_4-\Psi_3=\phi_{43}+\pi+\frac{\delta_{34}-\delta_{43}}{2},\\&\frac{\delta_{34}+\delta_{43}}{2}=\frac{\pi}{2}-\varepsilon_{34},\end{aligned}\right\}\tag{49}$$

又令

$$\left.\begin{aligned}&I_{21}=I_{21}^{(0)}+\widetilde{I}_{21},\quad I_{43}=I_{43}^{(0)}+\widetilde{I}_{43},\\&i_{21}=i_{21}^{(0)}+\widetilde{i}_{21},\quad i_{43}=i_{43}^{(0)}+\widetilde{i}_{43},\end{aligned}\right\}\tag{50}$$

其中 $I_{21}^{(0)}$、$I_{43}^{(0)}$、$i_{21}^{(0)}$、$i_{43}^{(0)}$ 满足略去 r_{12} 等后的方程 (44) 的定态解。再把"辐射捕获效应"加进方程式去[1],[11]以求更完善。最后方程(44)变换成：

$$\left.\begin{aligned}\frac{L}{C}\cdot\dot{\widetilde{I}}_{21}=&-I_{21}^{(0)}\left(\frac{\overline{\beta}_2}{\Gamma}+\overline{\vartheta}_{21}\right)\widetilde{I}_{21}-I_{21}^{(0)}\left[\left(\frac{1}{\Gamma}-1\right)\overline{\beta}_2+\overline{\vartheta}_A+\overline{\vartheta}_B\right]\widetilde{I}_{43}\\&+2I_{21}^{(0)}[\overline{r}_{21}\cos\varepsilon_{21}\cos\phi_{21}+\varDelta r_{21}\sin\varepsilon_{21}\sin\phi_{21}],\\\frac{L}{C}\dot{\widetilde{I}}_{43}=&(2\to4,\ 1\to3),\end{aligned}\right\}\tag{51-1}$$

$$\left.\begin{aligned}\frac{L}{C}\dot{\widetilde{i}}_{21}=&-I_{21}^{(0)}(\overline{\beta}_2-\overline{\vartheta}_{21})\widetilde{i}_{21}-I_{21}^{(0)}(\overline{\vartheta}_B-\overline{\vartheta}_A)\widetilde{i}_{43}\\&+2I_{21}^{(0)}[\overline{r}_{21}\sin\varepsilon_{21}\sin\phi_{21}-\varDelta r_{21}\cos\varepsilon_{21}\cos\phi_{21}],\\\frac{L}{C}\dot{\widetilde{i}}_{43}=&(2\to4,\ 1\to3),\end{aligned}\right\}\tag{51-2}$$

$$\begin{aligned}\dot{\phi}=\dot{\phi}_{21}-\dot{\phi}_{43}=&(\dot{\Psi}_2-\dot{\Psi}_1)-(\dot{\Psi}_4-\dot{\Psi}_3)\\=&\Omega+[2\overline{\rho}_2-2\overline{\tau}_{21}-(\tau_{31}+\tau_{42})+(\tau_{32}+\tau_{41})]\,\widetilde{i}_{21}\\&-[2\overline{\rho}_4-2\overline{\tau}_{43}-(\tau_{13}+\tau_{24})+(\tau_{23}+\tau_{14})]\,\widetilde{i}_{43}\\&-\Omega_{r21}\left[\cos\varepsilon_{21}\sin\phi_{21}+\frac{i_{21}^{(0)}}{I_{21}^{(0)}}\sin\varepsilon_{21}\cos\phi_{21}+\frac{\widetilde{i}_{21}}{I_{21}^{(0)}}\sin\varepsilon_{21}\cos\phi_{21}\right]\\&+\Omega_{r43}\cdot\left[\cos\varepsilon_{43}\sin\phi_{43}+\frac{i_{43}^{(0)}}{I_{34}^{(0)}}\sin\varepsilon_{43}\cos\phi_{43}+\frac{\widetilde{i}_{43}}{I_{43}^{(0)}}\sin\varepsilon_{43}\cos\phi_{43}\right],\end{aligned}\tag{51-3}$$

上述各式中：

$$\left.\begin{array}{ll}
\overline{\beta}_2 \equiv \dfrac{\beta_1+\beta_2}{2}, & \overline{\beta}_4 \equiv \dfrac{\beta_3+\beta_4}{2}, \\
\bar{\vartheta}_{21} \equiv \dfrac{\vartheta_{21}+\vartheta_{21}}{2}, & \bar{\vartheta}_{34} \equiv \dfrac{\vartheta_{34}+\vartheta_{43}}{2}, \\
\bar{\tau}_{21} \equiv \dfrac{\tau_{21}+\tau_{12}}{2}, & \bar{\tau}_{43} \equiv \dfrac{\tau_{43}+\tau_{34}}{2}, \\
\bar{r}_{21} \equiv \dfrac{r_{21}+r_{12}}{2}, & \bar{r}_{43} \equiv \dfrac{r_{43}+r_{34}}{2}, \\
\Delta r_{21} \equiv \dfrac{r_{21}-r_{12}}{2}, & \Delta r_{43} \equiv \dfrac{r_{43}-r_{34}}{2}, \\
\bar{\vartheta}_A \equiv \dfrac{\bar{\vartheta}_{41}+\bar{\vartheta}_{32}}{2}, & \bar{\vartheta}_B \equiv \dfrac{\bar{\vartheta}_{42}+\bar{\vartheta}_{31}}{2}, \\
\Delta\vartheta_A \equiv \dfrac{\bar{\vartheta}_{41}-\bar{\vartheta}_{32}}{2}, & \Delta\vartheta_B \equiv \dfrac{\bar{\vartheta}_{42}-\bar{\vartheta}_{31}}{2}, \\
\bar{\alpha}_2 \equiv \dfrac{\alpha_1+\alpha_2}{2}, & \bar{\alpha}_4 \equiv \dfrac{\alpha_3+\alpha_4}{2}, \\
\Delta\beta_2 \equiv \dfrac{\beta_2-\beta_1}{2}, & \Delta\beta_4 \equiv \dfrac{\beta_4-\beta_3}{2}, \\
\Delta\bar{\alpha}_2 \equiv \dfrac{\alpha_2-\alpha_1}{2}, & \Delta\bar{\alpha}_4 \equiv \dfrac{\alpha_4-\alpha_3}{2};
\end{array}\right\} \quad (52)$$

Γ是“辐射捕获效应”的参量[1]；$I_{21}^{(0)}$、$I_{43}^{(0)}$、$i_{21}^{(0)}$、$i_{43}^{(0)}$满足：

$$\left.\begin{array}{l}
\left(\dfrac{\dot{\beta}_2}{\Gamma}+\bar{\vartheta}_{21}\right) I_{21}^{(0)}+\left[\left(\dfrac{1}{\Gamma}-1\right)\overline{\beta}_2+\bar{\vartheta}_A+\bar{\vartheta}_B\right] I_{43}^{(0)}=\bar{\alpha}_2, \\
\left[\left(\dfrac{1}{\Gamma}-1\right)\overline{\beta}_4+\bar{\vartheta}_A+\bar{\vartheta}_B\right] I_{21}^{(0)}+\left(\dfrac{\overline{\beta}_4}{\Gamma}+\bar{\vartheta}_{43}\right) I_{43}^{(0)}=\bar{\alpha}_4,
\end{array}\right\} \quad (53\text{-}1)$$

$$\left.\begin{array}{l}
(\overline{\beta}_2-\bar{\vartheta}_{21})\, i_{21}^{(0)}+(\bar{\vartheta}_B-\bar{\vartheta}_A)\, i_{43}^{(0)}=\Delta\alpha_2-\dfrac{\Delta\beta_2}{\Gamma} I_{21}^{(0)} \\
\qquad -\left[\Delta\vartheta_B-\Delta\vartheta_A+\left(\dfrac{1}{\Gamma}-1\right)\Delta\beta_2\right] I_{43}^{(0)}, \\
(\bar{\vartheta}_B-\bar{\vartheta}_A)\, i_{21}^{(0)}+(\overline{\beta}_4-\bar{\vartheta}_{43})\, i_{43}^{(0)}=\Delta\alpha_4-\dfrac{\Delta\beta_4}{\Gamma} I_{43}^{(0)} \\
\qquad -\left[\Delta\vartheta_B+\Delta\vartheta_A+\left(\dfrac{1}{\Gamma}-1\right)\Delta\beta_4\right] I_{21}^{(0)},
\end{array}\right\} \quad (53\text{-}2)$$

(51-3)式中的Ω可写成：

$$\Omega=(1+SFC_{12})(\Omega_2-\Omega_1)-(1+SFC_{34})(\Omega_4-\Omega_3)+(SFC_{12}-SFC_{34})\omega_B$$
$$-(SFC_{12}+SFC_{34})\cdot 2Kv+\frac{C}{2L}f(\xi_{12},\xi_{34})\cdot(\gamma_2-\gamma_1), \quad (54\text{-}1)$$

其中

$$\left.\begin{aligned}SFC_{12}&\equiv\frac{G}{Ku}\cdot\frac{C}{2L}\cdot S(\xi_{12},\xi_{34}),\\SFC_{34}&\equiv\frac{G}{Ku}\cdot\frac{C}{2L}\cdot S(\xi_{34},\xi_{12}),\\f(\xi_{12},\xi_{34})&\equiv g(\xi_{12},\xi_{34})+g(\xi_{34},\xi_{12}).\end{aligned}\right\}\tag{54-2}$$

(54-1) 与近独立差动陀螺的数学形式几乎相同，只不过(54-2)式的各函数比近独立情形复杂得多！为省篇幅，不将各函数写下，对其性能待电子计算机算后作专文讨论。

还有，(51-3)式已用了

$$\varDelta r_{21}=\varDelta r_{43}=0,\tag{55}$$

这点和(51-1)、(51-2)不同，是作者一时疏忽而把它们抄在一起。不过，(55)式在以后总是假设的，因此，并不造成多少麻烦，说明一下就算了。(51-3)中还有两个量为：

$$\Omega_{r21}\equiv\frac{2C\bar{r}_{21}}{L},\qquad \Omega_{r43}\equiv\frac{2C\bar{r}_{43}}{L}.\tag{56}$$

由于(51-3)式是我们所需要的主要结果，其中并不包含$\widetilde{I_{21}}$、$\widetilde{I_{43}}$，故我们可以满足于讨论(51-1)式在一对称的特殊工作点下的解，此特例满足：

$$\bar{\alpha}_2=\bar{\alpha}_4=\bar{\alpha},\quad \bar{\beta}_2=\bar{\beta}_4=\bar{\beta},\quad \bar{\vartheta}_{21}=\bar{\vartheta}_{43}=\bar{\vartheta},\quad I^{(0)}_{21}=I^{(0)}_{43}=I^{(0)}.\tag{57}$$

又假设(55)式成立。这样，可得

$$\left.\begin{aligned}\frac{d}{dt}\cdot\frac{\widetilde{I_{21}}+\widetilde{I_{43}}}{2}&=-\Omega_{I+}\frac{\widetilde{I_{21}}+\widetilde{I_{43}}}{2}+\frac{I^{(0)}}{2}\cdot[\Omega_{r21}\cos\varepsilon_{21}\cos\phi_{21}\\&\quad+\Omega_{r43}\cos\varepsilon_{43}\cos\phi_{43}],\\\frac{d}{dt}\cdot\frac{\widetilde{I_{21}}-\widetilde{I_{43}}}{2}&=-\Omega_{I-}\frac{\widetilde{I_{21}}-\widetilde{I_{43}}}{2}+\frac{I^{(0)}}{2}\cdot[\Omega_{r21}\cos\varepsilon_{21}\cos\phi_{21}\\&\quad-\Omega_{r43}\cos\varepsilon_{43}\cos\phi_{43}],\end{aligned}\right\}\tag{58}$$

其中

$$\left.\begin{aligned}\Omega_{I+}&\equiv\frac{C\bar{\alpha}}{L},\\\Omega_{I-}&\equiv\frac{C\bar{\alpha}}{L}\cdot\frac{\Gamma(\bar{\beta}+\bar{\vartheta}-\bar{\vartheta}_B-\bar{\vartheta}_A)}{(2-\Gamma)\bar{\beta}+\Gamma(\bar{\vartheta}+\bar{\vartheta}_A+\bar{\vartheta}_B)},\end{aligned}\right\}\tag{59}$$

推导时用(53-1)以消去 $I^{(0)}$. 按[2]的“第二种近似法”的作法，(58)式的解为：

$$\left.\begin{aligned}\frac{\widetilde{I_{21}}+\widetilde{I_{43}}}{2}&=\frac{I^{(0)}}{2}\cdot\left[\Omega_{r21}\cos\varepsilon_{21}\cdot\frac{\Omega_{21}\sin\phi_{21}+\Omega_{I+}\cos\phi_{21}}{\Omega_{21}^2+\Omega_{I+}^2}\right.\\&\quad\left.+\Omega_{r43}\cos\varepsilon_{43}\cdot\frac{\Omega_{43}\sin\phi_{43}+\Omega_{I+}\cos\phi_{43}}{\Omega_{43}^2+\Omega_{I+}^2}\right],\\\frac{\widetilde{I_{21}}-\widetilde{I_{43}}}{2}&=\frac{I^{(0)}}{2}\cdot\left[\Omega_{r21}\cos\varepsilon_{21}\cdot\frac{\Omega_{21}\sin\phi_{21}+\Omega_{I-}\cos\phi_{21}}{\Omega_{21}^2+\Omega_{I-}^2}\right.\\&\quad\left.-\Omega_{r43}\cos\varepsilon_{43}\cdot\frac{\Omega_{43}\sin\phi_{43}+\Omega_{I-}\cos\phi_{43}}{\Omega_{43}^2+\Omega_{I-}^2}\right],\end{aligned}\right\}\tag{60}$$

其中 $\Omega_{21}\equiv\Omega_2-\Omega_1,\quad \Omega_{43}\equiv\Omega_4-\Omega_3.$ (61)

对(51-2)式的解就不应限于特例。把(51-2)重写成

$$\frac{d}{dt}\begin{pmatrix}\widetilde{i_{21}}\\ \widetilde{i_{43}}\end{pmatrix}=\frac{-C}{L}\begin{pmatrix}I_{21}^{(0)}, & 0\\ 0, & I_{43}^{(0)}\end{pmatrix}\begin{pmatrix}\overline{\beta}_2-\overline{\vartheta}_{21}, & \overline{\vartheta}_B-\overline{\vartheta}_A\\ \overline{\vartheta}_B-\overline{\vartheta}_A, & \overline{\beta}_4-\overline{\vartheta}_{43}\end{pmatrix}\begin{pmatrix}\widetilde{i_{21}}\\ \widetilde{i_{43}}\end{pmatrix}+\begin{pmatrix}I_{21}^{(0)}\Omega_{r21}\sin\varepsilon_{21}\sin\phi_{21}\\ I_{43}^{(0)}\Omega_{r43}\sin\varepsilon_{43}\sin\phi_{43}\end{pmatrix}. \tag{51-2'}$$

我们按标准方法求(51-2′)的解。先求左式矩阵的本征值及本征矢：

$$\frac{C}{L}\cdot\begin{pmatrix}I_{21}^{(0)}, & 0\\ 0, & I_{43}^{(0)}\end{pmatrix}\begin{pmatrix}\overline{\beta}_2-\overline{\vartheta}_{21}, & \overline{\vartheta}_B-\overline{\vartheta}_A\\ \overline{\vartheta}_B-\overline{\vartheta}_A, & \overline{\beta}_4-\overline{\vartheta}_{43}\end{pmatrix}\begin{pmatrix}a\\ b\end{pmatrix}=\Omega_i\begin{pmatrix}a\\ b\end{pmatrix}, \tag{62}$$

得

$$\left.\begin{aligned}&\Omega_i=\Omega_{i+},\qquad \Omega_{i-};\\ &\Omega_{i\pm}=\frac{C}{L}\left\{\frac{(\overline{\beta}_2-\overline{\vartheta}_{21})I_{21}^{(0)}+(\overline{\beta}_4-\overline{\vartheta}_{43})I_{43}^{(0)}}{2}\right.\\ &\left.\pm\frac{1}{2}\cdot\sqrt{[(\overline{\beta}_2-\overline{\vartheta}_{21})I_{21}^{(0)}-(\overline{\beta}_4-\overline{\vartheta}_{43})I_{43}^{(0)}]^2+4I_{21}^{(0)}I_{43}^{(0)}(\overline{\vartheta}_B-\overline{\vartheta}_A)^2}\right\},\end{aligned}\right\} \tag{63-1}$$

相应本征矢为 $\begin{pmatrix}a+\\ b+\end{pmatrix},\ \begin{pmatrix}a-\\ b-\end{pmatrix}.$ (63-2)

按标准方法，易证不同本征矢的正交性，并取归一化：

$$(a_k,b_k)\begin{pmatrix}I_{43}^{(0)}, & 0\\ 0, & I_{21}^{(0)}\end{pmatrix}\begin{pmatrix}a_{k'}\\ b_{k'}\end{pmatrix}=\delta_{k,k'},\ (k,k'=\pm) \tag{64}$$

用本征矢作展开：

$$\begin{pmatrix}\widetilde{i_{21}}\\ \widetilde{i_{43}}\end{pmatrix}=\widetilde{C}_+\begin{pmatrix}a+\\ b+\end{pmatrix}+\widetilde{C}_-\begin{pmatrix}a-\\ b-\end{pmatrix}, \tag{65}$$

代入(51-2′)，从(62)～(64)式得

$$\left.\begin{aligned}\dot{\widetilde{C}}_+&=-\Omega_{i+}\widetilde{C}_++I_{21}^{(0)}I_{43}^{(0)}(a_+\Omega_{r21}\sin\varepsilon_{21}\sin\phi_{21}\\ &\quad+b_+\Omega_{r43}\sin\varepsilon_{43}\sin\phi_{43}),\\ \dot{\widetilde{C}}_-&=-\Omega_{i-}\widetilde{C}_-+I_{21}^{(0)}I_{43}^{(0)}(a_-\Omega_{r21}\sin\varepsilon_{21}\sin\phi_{21}\\ &\quad+b_-\Omega_{r43}\sin\varepsilon_{43}\sin\phi_{43}).\end{aligned}\right\} \tag{66}$$

再按[2]的“第二种近似法”，得(66)式的解为

$$\widetilde{C}_\pm=I_{21}^{(0)}I_{43}^{(0)}\left\{\frac{a_\pm\Omega_{r21}\sin\varepsilon_{21}(\Omega_{i\pm}\sin\phi_{21}-\Omega_{21}\cos\phi_{21})}{\Omega_{21}^2+\Omega_{i\pm}^2}+\frac{b_\pm\Omega_{r43}\sin\varepsilon_{43}(\Omega_{i\pm}\sin\phi_{43}-\Omega_{43}\cos\phi_{43})}{\Omega_{43}^2+\Omega_{i\pm}^2}\right\}。 \tag{67}$$

除了($a_\pm$，$b_\pm$)未明显写出外，(67)式已是完整的解了。

(51-3)式中起“第二类闭锁效应”的项必含 $\phi_{21}-\phi_{43}$，根据 (67) 式，只有下面两项才是有关项：

$$\frac{\Omega_{r43}}{I_{43}^{(0)}}\cdot\sin\varepsilon_{43}(\widetilde{i_{43}\cos\phi_{43}})-\frac{\Omega_{r21}}{I_{21}^{(0)}}\cdot\sin\varepsilon_{21}\cdot(\widetilde{i_{21}\cos\phi_{21}}).\tag{68}$$

以(65)、(67)式代入(68)式，经过整理，可区分三种项：一种是高频振盪项可略去，如

$\sin\phi_{21},\cos\phi_{21},\sin2\phi_{21}$，$\cos2\phi_{21}$，$\sin2\phi_{43},\cdots\cdots$，$\cos(\phi_{21}+\phi_{43})$；

第二种是常数项对比例因子有贡献，如

$$\cos^2\phi_{21}=\frac{1}{2}+\frac{1}{2}\cos2\phi_{21}\text{ 的}\frac{1}{2};$$

第三种是含 $\phi_{21}-\phi_{43}\equiv\phi$ 的项，如

$$\left.\begin{aligned}\cos\phi_{21}\sin\phi_{43}&=-\frac{1}{2}\sin(\phi_{21}-\phi_{43})+\text{高频项},\\ \sin\phi_{21}\cos\phi_{43}&=\frac{1}{2}\sin(\phi_{21}-\phi_{43})+\text{高频项},\\ \cos\phi_{21}\cos\phi_{43}&=\frac{1}{2}\cos(\phi_{21}-\phi_{43})+\text{高频项。}\end{aligned}\right\}\tag{69}$$

因而(68)式去除高频项后，得有用部分

$$\begin{aligned}&\frac{I_{21}^{(0)}\Omega_{r43}\sin\varepsilon_{43}}{2}\cdot\sum_{k=\pm}\frac{a_kb_k\Omega_{r21}\sin\varepsilon_{21}[\Omega_{ik}\sin(\phi_{21}-\phi_{43})-\Omega_{21}\cos(\phi_{21}-\phi_{43})]}{\Omega_{i,k}^2+\Omega_{12}^2}\\ &+\frac{I_{43}^{(0)}\Omega_{r21}\sin\varepsilon_{21}}{2}\cdot\sum_{k=\pm}\frac{a_kb_k\Omega_{r43}\sin\varepsilon_{43}[\Omega_{ik}\sin(\phi_{21}-\phi_{43})+\Omega_{43}\cos(\phi_{21}-\phi_{43})]}{\Omega_{i,k}^2+\Omega_{43}^2}\\ &-\frac{I_{21}^{(0)}\Omega_{r43}^2\sin^2\varepsilon_{43}}{2}\sum_{k=\pm}\frac{b_k^2\Omega_{43}}{\Omega_{43}^2+\Omega_{ik}^2}+\frac{I_{43}^{(0)}\Omega_{r21}^2\sin^2\varepsilon_{21}}{2}\sum_{k=\pm}\frac{a_k^2\Omega_{21}}{\Omega_{21}^2+\Omega_{ik}^2}.\end{aligned}\tag{70}$$

上面推导中是使用了归一化条件(64)式的。如果本征矢不归一化，则 a_kb_k,a_k^2,b_k^2 皆应乘以归一化因子

$$\frac{1}{I_{43}^{(0)}a_k^2+I_{21}^{(0)}b_k^2}.\tag{71}$$

下面我们改取不归一化本征矢及归一化因子(71)较方便。回到本征方程(62)式，可取

$$\left.\begin{aligned}a_k&=-\frac{C}{L}\cdot(\bar\vartheta_B-\bar\vartheta_A)I_{21}^{(0)},\\ b_k&=\frac{C}{L}\left(\bar\beta_2-\bar\vartheta_{21}\right)I_{21}^{(0)}-\Omega_{i,k},\end{aligned}\right\}\tag{72}$$

其中$k=\pm$．用(63-1)的 $\Omega_{i,\pm}$ 值及(72)式，经过一些运算后得到

$$\frac{a_kb_k}{I_{43}^{(0)}a_k^2+I_{21}^{(0)}b_k^2}=\frac{C(\bar\vartheta_B-\bar\vartheta_A)}{L(\Omega_{ik}-\Omega_{i,-k})}.\tag{73}$$

代入(70)式，得：

$$(70)\text{ 式头两项}=\Omega_s\sin\phi+\Omega_c\cos\phi,\tag{74}$$

$$\Omega_s \cong \frac{C}{2L} \cdot \Omega_{r21}\Omega_{r43}\sin\varepsilon_{21}\sin\varepsilon_{43}(\bar{\vartheta}_B - \bar{\vartheta}_A)$$

$$\cdot\left[I_{21}^{(0)} \cdot \frac{\Omega_{21}^2 - \Omega_{i+}\Omega_{i-}}{(\Omega_{21}^2 + \Omega_{i+}^2)(\Omega_{21}^2 + \Omega_{i-}^2)} + I_{43}^{(0)} \cdot \frac{\Omega_{43}^2 - \Omega_{i+}\Omega_{i-}}{(\Omega_{43}^2 + \Omega_{i+}^2)(\Omega_{43}^2 + \Omega_{i-}^2)} \right], \tag{75-1}$$

$$\Omega_c \cong \frac{C}{2L} \cdot \Omega_{r21}\Omega_{r43}\sin\varepsilon_{21}\sin\varepsilon_{43}(\bar{\vartheta}_B - \bar{\vartheta}_A)$$

$$\cdot\left[I_{21}^{(0)} \cdot \frac{\Omega_{21}(\Omega_{i+} + \Omega_{i-})}{(\Omega_{21}^2 + \Omega_{i+}^2)(\Omega_{21}^2 + \Omega_{i-}^2)} - I_{43}^{(0)} \cdot \frac{\Omega_{43}(\Omega_{i+} + \Omega_{i-})}{(\Omega_{43}^2 + \Omega_{i+}^2)(\Omega_{43}^2 + \Omega_{i-}^2)} \right]; \tag{75-2}$$

(70)式的后两项

$$= (SFC'_{12} - SFC_{12})\Omega_{21} - (SFC'_{34} - SFC_{34})\Omega_{43}, \tag{76}$$

$$SFC'_{12} - SFC_{12} \cong \frac{1}{4} \sum_{k=\pm} \frac{\Omega_{r21}^2 \sin^2\varepsilon_{21}}{\Omega_{21}^2 + \Omega_{ik}^2}$$

$$+ \frac{C^2\Omega_{r21}^2 \sin^2\varepsilon_{21}[I_{43}^{(0)2}(\bar{\beta}_4 - \bar{\vartheta}_{43})^2 - I_{21}^{(0)2}(\bar{\beta}_2 - \bar{\vartheta}_{21})^2]}{4L^2(\Omega_{21}^2 + \Omega_{i+}^2)(\Omega_{21}^2 + \Omega_{i-}^2)}, \tag{77-1}$$

$$SFC'_{34} - SFC_{34} \cong \frac{1}{4} \sum_{k=\pm} \frac{\Omega_{r43}^2 \sin^2\varepsilon_{43}}{\Omega_{43}^2 + \Omega_{ik}^2}$$

$$+ \frac{C^2\Omega_{r43}^2 \sin^2\varepsilon_{43}[I_{21}^{(0)2}(\bar{\beta}_2 - \bar{\vartheta}_{21})^2 - I_{43}^{(0)2}(\bar{\beta}_4 - \bar{\vartheta}_{43})^2]}{4L^2(\Omega_{43}^2 + \Omega_{i+}^2)(\Omega_{43}^2 + \Omega_{i-}^2)}. \tag{77-2}$$

因此，把上面结果代入(51-3)式后，去除高频振盪项，便得到

$$\dot{\phi} \cong \dot{\phi}_{21} - \dot{\phi}_{43} = \Omega' + \Omega_s \sin\phi + \Omega_c \cos\phi, \tag{78}$$

式中

$$\Omega' = (54\text{-}1\text{式的}\Omega)_{SFC_{12} \to SFC'_{12},\, SFC_{34} \to SFC'_{34}}. \tag{79}$$

Ω' 与 Ω 的差别在于 (77-1)、(77-2)的量，这正是[2]所详细讨论的正比例因子修正项在差动陀螺情形下的推广。

情形 1 低偏频量，满足

$$\Omega_{21}^2 \ll \Omega_{i\pm}^2,\quad \Omega_{43}^2 \ll \Omega_{i\pm}^2. \tag{80}$$

则(75-1)、(75-2)可简化为

$$\left.\begin{aligned} \Omega_s &\cong -\frac{C}{2L} \cdot \frac{\Omega_{r21}\Omega_{r43}}{\Omega_{i+}\Omega_{i-}} \cdot \sin\varepsilon_{21}\sin\varepsilon_{43}(I_{21}^{(0)} + I_{43}^{(0)}) \cdot (\bar{\vartheta}_B - \bar{\vartheta}_A), \\ \Omega_c &\cong -\Omega_s \cdot \frac{I_{21}^{(0)}\Omega_{21} - I_{43}^{(0)}\Omega_{43}}{I_{21}^{(0)} + I_{43}^{(0)}} \cdot \left(\frac{1}{\Omega_{i+}} + \frac{1}{\Omega_{i-}}\right) \cong 0. \end{aligned}\right\} \tag{81}$$

情形 2 高偏频量，满足

$$\Omega_{21}^2 \gg \Omega_{i\pm}^2,\quad \Omega_{43}^2 \gg \Omega_{i\pm}^2. \tag{82}$$

则

$$\left.\begin{aligned}\Omega_s&\cong\frac{C}{2L}\cdot\sin\varepsilon_{21}\sin\varepsilon_{43}\cdot\Omega_{r21}\Omega_{r43}\\&\quad\cdot\left(\frac{I_{21}^{(0)}}{\Omega_{21}^2}+\frac{I_{43}^{(0)}}{\Omega_{43}^2}\right)\cdot(\bar\vartheta_B-\bar\vartheta_A),\\\Omega_c&\cong\Omega_s\cdot(\Omega_{i+}+\Omega_{i-})\cdot\left(\frac{I_{21}^{(0)}}{\Omega_{21}^3}-\frac{I_{43}^{(0)}}{\Omega_{43}^3}\right)\Bigg/\left(\frac{I_{21}^{(0)}}{\Omega_{21}^2}+\frac{I_{43}^{(0)}}{\Omega_{43}^2}\right)\cong0.\end{aligned}\right\}\tag{83}$$

四频差动陀螺使用$Ne^{20}:Ne^{22}\cong53:47$，它们的中心圆频平分别为$\omega_0$和$\omega_0'$，有[1]

$$\left.\begin{aligned}&\frac{\omega_0'-\omega_0}{Ku}=0.880,\\&\xi_j=\frac{\omega_j-\omega_0}{Ku},\ \ \xi_j'=\sqrt{1.1}\cdot\frac{\omega_j-\omega_0'}{Ku}=\sqrt{1.1}\,(\xi_j-0.880);\end{aligned}\right\}\tag{84}$$

(39)、(40)式各系数应按比例混合才是真正使用的值，如

$$\vartheta=F\cdot\vartheta_{Ne}^{20}+(1-F)\cdot\sqrt{1.1}\vartheta_{Ne}^{22},\ \text{等},\tag{85}$$

$F\cong0.53$.

下面算一点数字：从[12]

$$\frac{\gamma_a}{2\pi}=10+3.3p,\ \frac{\gamma_b}{2\pi}=13+13p,\ \frac{\gamma}{2\pi}=10+57p,\ MHz\tag{86}$$

p的单位用乇。又

$$\eta_a=\frac{\gamma_a}{Ku},\qquad\eta_b=\frac{\gamma_b}{Ku},\qquad\eta=\frac{\gamma}{Ku}.\tag{87}$$

算得(41)式的系数列于表2.

$$a=\left(\frac{21}{\eta_a}+\frac{1}{\eta_b}\right)\Bigg/\left[46\left(\frac{1}{\eta_a}+\frac{1}{\eta_b}\right)\right]$$

表 2

p，乇	0	0.5	1	1.5	2	2.5	3	3.5	4
a	0.2675	0.2939	0.3094	0.3195	0.3267	0.3321	0.3362	0.3395	0.3422

在常用范围，根据表2，可取$a=\frac{1}{3}$. 按(52)、(85)、(39)诸式可算得对称工作点(即$\xi_{12}+\xi_{34}=0.880$)时的ϑ_A、ϑ_B(当然，39式只包括烧孔效应项，忽略粒子数脈动项)，见表3. 计算时，L取500mm，即$\frac{C}{2L}$取300MHz.

$\bar{\vartheta}_A$，$\bar{\vartheta}_B\left(当\dfrac{C}{2L}=300MHz,\dfrac{\xi_{12}+\xi_{34}}{2}=0.440\right)$ **表 3**

η	0.05	0.1	0.2
$\bar{\vartheta}_A/G$	0.02900	0.09405	0.21430
$\bar{\vartheta}_B/G$	0.01123	0.04604	0.17809
$(\bar{\vartheta}_A-\bar{\vartheta}_B)/G$	0.01777	0.04801	0.03621

数字例 1*

$$\left.\begin{aligned}&\frac{\Omega_{r,21}}{2\pi}\cong\frac{\Omega_{r,43}}{2\pi}\cong 0.5\text{度/秒},\ \sin\varepsilon_{21}\cong\sin\varepsilon_{43}\cong 1,\\&\frac{\Omega_{i\pm}}{2\pi}\cong 5\text{ 度/秒},\quad \frac{\Omega_{21}}{2\pi}\cong\frac{\Omega_{43}}{2\pi}\cong 1\text{度/秒},\\&\frac{C}{2L}=300MHz,\quad G\cong 0.02,\quad \eta=0.2,\\&I_{21}^{(0)}\cong I_{43}^{(0)}\cong 5\times 10^{-3},\ \text{比例因子}1.5Hz/(\text{度/时})。\end{aligned}\right\}\tag{88}$$

本例符合(80)式条件，故从(81)式可得

$$\Omega_s/2\pi\cong 14\text{度/时},\ \Omega_c/2\pi\cong 0.\tag{89}$$

数字例 2

$$\frac{\Omega_{r21}}{2\pi}\cong\frac{\Omega_{r43}}{2\pi}\cong 2\text{度/秒，其余同例 1}.\tag{90}$$

则 $\Omega_s/2\pi\cong 16\times 14\cong 220\text{度/时},\ \Omega_c/2\pi\cong 0.$ (91)

(91)式的数字与我们 79 年末～80 年初的实验数据(≈200 度/时) 基本一致。

数字例 3

$$\frac{\Omega_{21}}{2\pi}\cong\frac{\Omega_{43}}{2\pi}\cong 200\text{度/秒，其余同例 2}.\tag{92}$$

则符合(82)式条件，故从(83)式可得

$$-\Omega_s/2\pi\cong 0.14\text{度/时},\ \Omega_c/2\pi/\cong 0.\tag{93}$$

(93)式的数字和我们观察值相近。

实验还表明：当元件质量好时，正常测试时观察不到第二类闭锁效应。这是容易理解的，請看例4.

数字例 4

$$\frac{\Omega_{21}}{2\pi}\cong\frac{\Omega_{43}}{2\pi}\cong 200\text{度/秒，其余同例 1}.\tag{94}$$

则 $-\Omega_s/2\pi\cong 0.01\text{ 度/时},\ \Omega_c/2\pi\cong 0.$ (95)

这样，在实验精度内就观察不到此效应了。

* 数字的来源见[2]，$I^{(0)}$的来源则见[1]。

由于锁区的大小与$\bar{\vartheta}_B-\bar{\vartheta}_A$成正比，我们感兴趣的是如何使该值为0．由数字计算，至少在对称工作点附近

$$\left(\frac{\xi_{21}+\xi_{43}}{2}=0.440\right)，准到\ 2\%\ 精度内$$

$$\bar{\vartheta}_A/(a\bar{\vartheta}_B)=\mathcal{L}\left(\frac{\xi_{21}-\xi_{43}}{2}\right)\Big/\mathcal{L}\left(\frac{\xi_{21}+\xi_{43}}{2}\right), \tag{96}$$

其中$\mathcal{L}$函数见(34)式。当取

$$\bar{\vartheta}_A=\bar{\vartheta}_B,\quad a=\frac{1}{3},\qquad \frac{\xi_{21}+\xi_{43}}{2}=0.440, \tag{97}$$

则得

$$(\xi_{21}-\xi_{43})^2=0.25813-\frac{8\eta^2}{3}. \tag{98}$$

就是说：只要满足(98)式，则$\bar{\vartheta}_B-\bar{\vartheta}_A=0$，$\Omega_s=\Omega_c=0$. 实际设计时应向这个指标靠拢。数字见表4.

$\vec{\vartheta}_A=\vec{\vartheta}_B$ 的条件　　表 4

η	0.05	0.1	0.15	0.2	0.25	0.3
$\xi_{43}-\xi_{21}$	0.5015	0.4811	0.4450	0.3892	0.3024	0.1437

如果水晶片旋光角为90°，则

$$|\xi_{43}-\xi_{21}|=\frac{C/2L}{1000MHz}=\begin{cases}0.385 & 当\ L=390\text{mm}; \\ 0.313 & 当\ L=480\text{mm};\end{cases} \tag{99}$$

对照表4，应使用$\eta=0.2$，0.24.

如果η与$|\xi_{43}-\xi_{21}|$的关系偏离(98)式或表4的关系，则$\vec{\vartheta}_A/\vec{\vartheta}_B$偏离1．由(96)式，易得两种偏离值的关系为：

$$\delta(\vec{\vartheta}_A/\vec{\vartheta}_B)=3.72\delta\eta. \tag{100}$$

如果要此值小于0.02（绝对值），则$|\delta\eta|\leqslant0.0054$，相当于总充气压准到0.1毛，已达极限。并且，经验公式(86)的误差尚未计及，需要在实验过程中加以矫正。总之，如不作巨大努力，目前$\bar{\vartheta}_A-\bar{\vartheta}_B$的值基本上属于表3的量级。

结　论

一、本文是对四频差动陀螺左旋模与右旋模之间的耦合效应的第一个计算，得出第二类闭锁效应的定量公式（当然，还有正比例因子），在性质上与数量上皆与我们自己近年的实验相符，确认了此效应。

二、要第二类闭锁效应小。必须

(1)Ω_r小($\because\propto\Omega_{r21}\Omega_{r43}$)，即元件非均匀性小；

(2)偏频量大$\left(\because\propto\frac{1}{\Omega_{21}\Omega_{43}}\right)$;

(3)光强弱($\because\propto I_{21}^{(0)}$及$I_{43}^{(0)}$);

(4) $|\bar{\vartheta}_A-\bar{\vartheta}_B|$小($\because\propto|\bar{\vartheta}_A-\bar{\vartheta}_B|$);

(5)$|\sin\varepsilon_{21}\sin\varepsilon_{43}|$小.

实际上我们能控制(1)、(2)因素，可能还能控制因素(4)。

对光强削弱则有副作用，它使量子噪声大，单陀螺的闭锁阈值大[2]以及输出信号弱。故光强只能适当取值。至于ε_{21}、ε_{43}的控制，到现在尚毫无所知。

三、在目前最好的元件及已达到偏频量的条件下，第二类闭锁效应已可以不影响精度。甚至已可以观察不到（参见数字例4）。

补充一下：我们用过质量较差的硬膜，它们在大偏频量时亦曾形成数百度/时的闭锁效应。这是反面的教训。

四、经过巨大努力，我们有可能接近$\bar{\vartheta}_A=\bar{\vartheta}_B$，这样第二类闭锁效应就更小，甚至使用质量较次的元件亦不妨。

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The Locking Phenomenon of Second kind in Differential Laser Gyro

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Abstract

This paper considers the coupling effects between the left and right handed circularly polarized modes, and obtains fundamental formulas. Under the special conditions that the couplings between the same handed palarized modes are the chief ones, we oftain an approximate analytic solution, quantitatively describing the locking phenomenon of second kind, which show, that every simple laser gyro lies in travelling wave state, while the differential beat frequency locks. The results coincide with our experimental data in recent years. We point out also the effective ways to weaken it. Besides, we oftain the positive terms of relative scale factor correction, which are the generalization of corresponding terms in simple laser gyro.