$$G_2 = B_1 - B_2 D_{12}^{+} D_{11} \tag{63.b}$$

$$L_a = K_a^{\top} D_{12}^{\perp} + R_a G_1 \tag{63.c}$$

$$M_a = - R_a B_1 + L_a D_{11} \tag{63.d}$$

$$L_b = K_b^{\top} D_{21}^{\perp\top} + \hat{R}_b^{\top} C_2^{\top} D_{21}^{+\top} \tag{63.e}$$

$$M_b = - \hat{R}_b^{\top} C_1^{\top} + L_b D_{11}^{\top} \tag{63.f}$$

$$D_0 = \begin{bmatrix} D_{12}^{\perp\top} & D_{11} D_{21}^{\top} \\ D_{11}^{\top} D_{12}^{\perp\top} & D_{21}^{\perp} \end{bmatrix} \tag{63.g}$$

4 Conclusion

In this paper we have solved the control problem for linear system with part uncertain parameters under bounded noise. The effect of parameters uncertainty can be converted to an equivalent effect of a new set of bounded noise. Thus above problem can be converted into a H_∞ control problem of a scaled difinite system under an extended bounded noise. Then we have dealt with the H_∞ standard problem of four blocks in general case ($D_{22} \neq 0$), and further obtained the explicit formulas of complete solution set for the parameter robust control problem. These formulas are related to the contolled plant parameters and design parameters directly, and can be programmed on computer conveniently.

In summary, this paper is concerned with parameter robust control problem which appears in many engineering design tasks, and gives the explicit formulas of complete solution set. These results will be with a good prospect in practical engineering applications.

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部分参数不确定线性系统的鲁棒控制

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摘　要　本文研究了部分参数不确定的线性系统在有界能量噪声作用下的控制问题，即参数鲁棒性控制问题。具有准确模型的飞机在不准确气动参数信息条件下的飞行控制就是一个典型例子。我们先把它简化为一个带正参数的 H_∞ 控制问题，然后采用 J 无损分解方法，推导了调节问题的可解性条件和动态反馈控制器的全部显式通解。

关键词：参数鲁棒性控制，H_∞ 标准控制问题，J 无损分解

分类号：TP13

国防科技大学学报
JOURNAL OF NATIONAL UNIVERSITY OF DEFENSE TECHNOLOGY
第16卷第1期 1994年3月 Vol. 16 No. 1

Robust Control of Linear Systems with Part Uncertain Parameters

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Abstract This paper is concerned with the control problem of linear systems with part uncertain parameters under a bounded energy noise--the parameter robust control. The flight control of airplane with a precise model under an uncertain information condition of aerodynamic parameters, is a typical example. At first we simplify it into a scaled H_∞ control problem, and then deduce a solvability condition and give an explicit cxpression of the complete solution set for the parameter robust control problem by J-lossless factorization method.

Key words Parameter robust control, H_∞ standard control problem, J-lossless factorization

1 Introduction

If all parameters of a linear system can be known precisely, the control problems of it under noise with a given spectral density are well studied in LQG criterion, and that can be solved by the separation principle.

The control problems of above system under a bounded energy noise with unknown spectral density, is discribed in H_∞ framework. On the other hand, for a definite system with a model varying in an uncertainty range described by H_∞ norm, it's robust stability problem is a typical H_∞ problem. Recently ten years researches have given complete solutions for these two H_∞ problems.

The control problem of a plant with H_∞ uncertain model under bounded energy noises with unknown spectral density, is a μ problem proposed by Doyle. Until now how to solving μ problem is still open. Thus it does not been concerned with in this paper.

This paper is concerned with another common uncertainty—parameter uncertainty. In this case the framework and order of plant's model is known precisely and correctly, but the parameters in model are not precisely known, and they may vary in some real in-

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* Received June 15, 1993

tervals. A more generalized statement is，some part parameters are known accurately，and other part parameters may vary in some real intervals. For example，in the flight process of an airplane，the parameters of airplane self can be given accurately，but the aerodynamic parameters may be with a quite large uncertainty and vary in given ranges. Although Kharitonov proposed a convenient criterion to judge if all the roots of the interval coefficiant polynomial are stable，it seems helpless to this problem：How to design a compensator for the uncertain parameter plant under bounded energy noise of unknown spectral density[1]. In this paper we will convert it into a scaled H_∞ control problem，and then give an explicit expression for complete solution set for this parameter robust control problem by J-lossless factorization method.

2 Problem Formulation

Consider the following uncertain system P：

$$\dot{X}(t) = (A + \Delta A(t))X(t) + (B + \Delta B(t))U(t) + B_wW(t) \qquad (1.a)$$

$$Y(t) = (C + \Delta C(t))X(t) + (D + \Delta D(t))U(t) + D_wW(t) \qquad (1.b)$$

$$Z(t) = C_z(t)X(t) + D_z(t)U(t) \qquad (1.c)$$

where $X(t)\in R^n$ is the state，$U(t)\in R^k$ is control input，$W(t)\in R^m$ is the noise，$Y(t)\in R^r$ is the measured output，$Z(t)\in R^p$ is the controlled output. A、B、C、D、B_w、C_z、D_z and D_w are given real constant matrices of dimensions of $n\times n$、$n\times k$、$r\times n$、$r\times k$、$n\times m$、$p\times n$、$p\times k$、$r\times m$. $\Delta A(t)$、$\Delta B(t)$、$\Delta C(t)$ and $\Delta D(t)$ are real-valued matrix functions which represent time-varying parameter uncertainties. The parameter uncertainties considered here are norm-bounded and of following forms：

$$\begin{bmatrix}\Delta A(t) & \Delta B(t)\\ \Delta C(t) & \Delta D(t)\end{bmatrix} = \begin{bmatrix}H_1\\ H_2\end{bmatrix}F(t)[E_1 \ \ E_1] \qquad (2)$$

where H_1、H_2、E_1、and E_2 are known constant matrices of dimensions of $n\times b_1$、$r\times b_1$、$b_2\times n$、$b_2\times k$ respectively. $F(t)\in R^{b_1\times b_2}$ is an unknown matrix function in unit ball：

$$F^T(t)F(t) \leqslant I \qquad \forall\ t \qquad (3)$$

We need to design a compensator C：

$$\dot{X}_e(t) = A_eX_e(t) + B_eY(t)，\ X_e(0) = 0 \qquad (4.a)$$

$$U(t) = K_eX_e(t)， \qquad (4.b)$$

such that the transfer function $T(s)$ from noise W to controlled output Z to satisfy the requirement

$$\|T(s)\|_\infty \leqslant \gamma \qquad (5)$$

where γ is a small positive real.

The closed loop system composed of P and C can be written in a compact form

$$\dot{\xi} = (A_c + H_cFE_c)\xi + B_cW \qquad (6.a)$$

$$Z = C_c\xi \qquad (6.b)$$

where

$$\xi = \begin{bmatrix} X \\ X_c \end{bmatrix} \tag{7}$$

$$A_c = \begin{bmatrix} A & BK_c \\ B_cC & A_c + B_cDK_c \end{bmatrix} \tag{8.a}$$

$$B_c = \begin{bmatrix} B_w \\ B_cD_w \end{bmatrix} \tag{8.b}$$

$$C_c = [C_1 \quad D_zK_c] \tag{8.c}$$

$$H_c = \begin{bmatrix} H_1 \\ B_cH_2 \end{bmatrix} \tag{8.d}$$

$$E_c = [E_1 \quad E_2K_c] \tag{8.e}$$

Thus the transfer function from W to Z is

$$T(s) = C_c[sI - (A_c + H_cFE_c)]^{-1}B_c \tag{9}$$

The fact of plant P and closed loop transfer function $T(s)$ containing an unknown F, causes many difficulties for further handling. To overcome it, we can consider a plant $\bar{P}_\delta$ with a positive real δ:

$$\dot{\bar{X}} = A\bar{X} + BU + [B_w \quad \gamma\delta^{-1}H_1]\bar{W} \tag{10.a}$$

$$\bar{Y} = C\bar{X} + DU + [D_w \quad \gamma\delta^{-1}H_2]\bar{W} \tag{10.b}$$

$$\bar{Z} = \begin{bmatrix} C_z \\ \delta E_1 \end{bmatrix}\bar{X} + \begin{bmatrix} D_z \\ \delta E_2 \end{bmatrix}U \tag{10.c}$$

$\bar{P}_\delta$ will become the same as P when putting

$$\bar{W} = \begin{bmatrix} W \\ \delta\gamma^{-1}F(E_1\bar{X} + E_2U) \end{bmatrix} \tag{11}$$

in (10), but it is helpless for solving the problem. Therefore we will consider $\bar{W}$ as a bounded noise, and will not restrict $\bar{W}$ in (11) form.

Applying the compensator C to the plant $\bar{P}_\delta$, the closed loop system composed of (4) and (10) can be written as

$$\dot{\bar{\xi}} = A_c\bar{\xi} + \bar{B}_c\bar{W} \tag{12.a}$$

$$\bar{Z} = \bar{C}_c\bar{\xi} \tag{12.b}$$

where

$$\bar{\xi} = \begin{bmatrix} \bar{X} \\ X_c \end{bmatrix} \tag{13.a}$$

$$\bar{B}_c = [B_c \quad \gamma\delta^{-1}H_c] \tag{13.b}$$

$$\bar{C}_c = \begin{bmatrix} C_c \\ \delta E_c \end{bmatrix} \tag{13.c}$$

The transfer function from noise $\bar{W}$ to extended controlled output $\bar{Z}$ is

$$\bar{T}_\delta(s) = \bar{C}_c(sI - A_c)^{-1}\bar{B}_c \tag{14}$$

Lemma 1

Linear system

$$R(s) = C(sI - A)^{-1}B \tag{15}$$

satisfies H_∞ norm inequality

$$\|R(s)\|_\infty \leqslant \gamma, \tag{16}$$

If and only if there exists a positive symmetric matrix X satisfying Riccati inequality

$$A^TX + XA + C^TC + \gamma^{-2}XBB^TX \leqslant 0 \tag{17}$$

Proof. It is from the book of Francis[2].

Theorem 1

If there exists a positive real δ and a compensator C for the scaled plant $\overline{P}_\delta$, such that the Transfer function $\overline{T}_\delta(s)$ of closed loop composed of $\overline{P}_\delta$ and C satisfies

$$\|\overline{T}_\delta(s)\|_\infty \leqslant \gamma \tag{18}$$

then the transfer fumction $T(s)$ of closed loop composed of P and C will satisfy

$$\|T(s)\|_\infty \leqslant \gamma \tag{19}$$

Proof. If (18) exists, applying lemmal to (18), There exists a positive symmtric matrix X satisfying

$$A_c^TX + XA_c + \begin{bmatrix} C_c \\ \delta E_c \end{bmatrix}^T \begin{bmatrix} C_c \\ \delta E_c \end{bmatrix} + \gamma^{-2}X[B_c \quad \gamma\delta^{-1}H_c][B_c \quad \gamma\delta^{-1}H_c]^TX \leqslant 0 \tag{20}$$

Now notice

$$E_c^TF^TH_c^TX + XH_cFE_c \leqslant \delta^{-2}XH_cH_c^TX + \delta^2E_c^TF^TFE_c \leqslant \delta^{-2}XH_cH_c^TX + \delta^2E_c^TE_c \tag{21}$$

then we have

$$(A_c + H_cFE_c)^TX + X(A_c + H_cFE_c) + C_c^TC_c + \gamma^{-2}XB_cB_c^TX = 0 \tag{22}$$

Then applying lemmal agaim, (19) is proven.

3 Complete solution set of the problem

Theorem 1 tells us that the solvability of regulation problem (18) of scaled plant $\overline{P}_\delta$ can guarantee the regulation of uncertain plant P to satisfying (19). In the scaled plant $\overline{P}_\delta$, there is no any uncertain parameters except an artificial scale parameter δ. Thus we can solve problem (18) in the framework of H_∞ standard problem. To normalize the problem, we introduce a new noise

$$\widetilde{W} = \gamma\overline{W} \tag{23}$$

the closed loop transfer function from $\widetilde{W}$ to $\overline{Z}$ is

$$T_\delta(s) = \gamma^{-1}T(s) \tag{24}$$

Thus the problem (18) becomes a normalized problem

$$\|T_\delta(s)\|_\infty \leqslant 1 \tag{25}$$

It can be put into the framework of H_∞ standard problem.

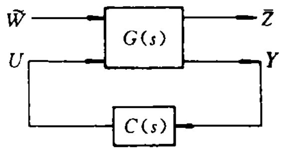

Fig 1 Generalized plant of H_∞ standard problem

The expression in state space of transfer function of generalized plant from $\begin{bmatrix}\widetilde{W}\\U\end{bmatrix}$ to $\begin{bmatrix}\bar{Z}\\Y\end{bmatrix}$ is

$$G(s)=\left\{A,\ [B_1\ \ B_2],\ \begin{bmatrix}C_1\\C_2\end{bmatrix},\ \begin{bmatrix}D_{11} & D_{12}\\D_{21} & D_{22}\end{bmatrix}\right\} \tag{26}$$

where

$$B_1=[\gamma^{-1}B_w\quad \delta^{-1}H_1] \tag{27.a}$$

$$B_2=B \tag{27.b}$$

$$C_1=\begin{bmatrix}C_z\\\delta E_1\end{bmatrix} \tag{27.c}$$

$$C_2=C \tag{27.d}$$

$$D_{11}=0 \tag{27.e}$$

$$D_{12}=\begin{bmatrix}D_z\\\delta E_2\end{bmatrix} \tag{27.f}$$

$$D_{21}=[\gamma^{-1}D_w\quad \delta^{-1}H_2] \tag{27.g}$$

$$D_{22}=D \tag{27.h}$$

Suppose $(A,\ B_2)$ is stabilizable and $(C_2,\ A)$ is detectable. There exist matrices M and N such that

$$A_m=A+B_2M \tag{28.a}$$

$$A_n=A+NC_2 \tag{28.b}$$

are stable. The Youla free parameter Q is related to the compensator C is following way

$$C(s)=(Z_{11}Q+Z_{12})(Z_{21}Q+Z_{22})^{-1},\ Q\in H_\infty^{k\times r} \tag{29}$$

and that causes $T_\delta(s)$ internal stable, where

$$Z(s)=\begin{bmatrix}Z_{11} & Z_{12}\\Z_{21} & Z_{22}\end{bmatrix}=\left\{A_m,\ -[B_2\ \ N],\ \begin{bmatrix}M\\C_2+D_{22}M\end{bmatrix},\ \begin{bmatrix}-I_k & 0\\-D_{22} & I_r\end{bmatrix}\right\} \tag{30}$$

Now (24) becomes

$$T_\delta(s)=T_1(s)-T_2(s)Q(s)T_3(s) \tag{31}$$

and (25) becomes a normalized model-Matching problem

$$\|T_1(s)-T_2(s)Q(s)T_3(s)\|_\infty\leqslant 1,\ Q\in H_\infty^{k\times r}, \tag{32}$$

The expressions in state space of T_1, T_2, T_3 are

$$\begin{bmatrix} T_1 & T_2 \\ T_3 & 0 \end{bmatrix} = \left\{ \begin{bmatrix} A + B_2M & -B_2M \\ 0 & A + NC_2 \end{bmatrix}, \begin{bmatrix} B_1 & B_2 \\ B_1 + ND_{21} & 0 \end{bmatrix}, \right.$$

$$\left. \begin{bmatrix} C_1 + D_{12}M & -D_{12}M \\ 0 & C_2 \end{bmatrix}, \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & 0 \end{bmatrix} \right\} \tag{33}$$

The complete solutions of the normalized model matching problem (32) can be figured out in following way. First try to find a normalized left annihilator $V_a(s)$ of $T_2(s)$, and a normalized right annihilator $V_b(s)$ of $T_3(s)$. They will be used to construct (J, J') unitary matrix

$$\Theta_0 = \begin{bmatrix} \tilde{V}_a & T_1V_b \\ \tilde{T}_1\tilde{V}_a & V_b \end{bmatrix} \tag{34}$$

to satisfy the (J, J') unitary relation

$$\tilde{\Theta}_0 J \Theta_0 = J' \tag{35}$$

then try to find Θ_1, a J-orthogonal complement of Θ_0. And then try to find unimodular matrix Π_a from the basic relation

$$\begin{bmatrix} -T_2 & 0 \\ 0 & I \end{bmatrix} \Pi_a = \begin{bmatrix} I & -T_1 \\ 0 & T_3 \end{bmatrix} \Theta_1 \tag{36}$$

where unimodular matrix means that all the poles and zeroes of the matrix are in left half plane. (36) is called J-lossless factorization. Divide the unimodular matrix Π_a into four blocks properly,

$$\Pi_a = \begin{array}{cc} \begin{bmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{21} & \Pi_{22} \end{bmatrix} & \begin{array}{c} k \\ r \end{array} \\ \begin{array}{cc} k & \quad r \end{array} & \end{array} \tag{37}$$

and also divied Θ_1 into four blocks properly

$$\Theta_1 = \begin{array}{cc} \begin{bmatrix} \theta_{11} & \theta_{12} \\ \theta_{21} & \theta_{22} \end{bmatrix} & \begin{array}{c} p+b_2 \\ m+b_1 \end{array} \\ \begin{array}{cc} k & \quad r \end{array} & \end{array} \tag{38}$$

then the solution set of normalized model matching problem (32) is

$$Q = (\Pi_{11}\Phi + \Pi_{12})\ (\Pi_{21}\Phi + \Pi_{22})^{-1},\ \Phi \in BH_\infty^{k \times r} \tag{39}$$

and the result transfer function of regulated error is

$$\overline{T}_\delta(s) = \gamma \cdot [\theta_{11}\Phi + \theta_{12}\ \ T_1V_b][\theta_{21}\Phi + \theta_{22}\ \ V_b]^{-1} \tag{40}$$

From (29), we can define a matrix $L = Z\Pi_a$, and divide it into four blocks properly

$$L = Z\Pi_a = \begin{array}{cc} \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} & \begin{array}{c} k \\ r \end{array} \\ \begin{array}{cc} k & \quad r \end{array} & \end{array} \tag{41}$$

At last we will find that the expression of solution set for the compensator C is

$$C = (L_{11}\Phi + L_{12})(L_{21}\Phi + L_{22})^{-1},\ \Phi \in BH_\infty^{k \times r} \tag{42}$$

Now we are going to calculate all of these cxplicitly according to above statement. The result of Kimura, Lu and Kawatani[3] on H_∞ standard control problem is a nice reference for our calculation, but they only consider the case of $D_{22}=0$ for simplicity. In our problem we must consider a more general case since in (27.h) $D_{22}=D\neq 0$. Our results also reveal several mistakes of [3]. We will only list results at maim steps since the calculation is evey cumbersome.

Choose a left inverse D_{12}^{+} and a left annihilator $D_{12}^{\perp}$ of D_{12} to satisfy

$$\begin{bmatrix} D_{12}^{+} \\ D_{12}^{\perp} \end{bmatrix} [D_{12} \quad (I - D_{11}D_{11}^{\top})D_{12}^{\perp\top}] = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}. \tag{43}$$

and choose a right inverse D_{21}^{+} and a right annihilator $D_{21}^{\perp}$ of D_{21} to satisfy

$$\begin{bmatrix} D_{21} \\ D_{21}^{\perp\top}(I - D_{11}^{\top}D_{11}) \end{bmatrix} [D_{21}^{+} \quad D_{21}^{\perp}] = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \tag{44}$$

Next step is to find the normalized left annihilator $V_a(s)$. Define

$$A_a = A - B_2D_{12}^{+}C_1 \tag{45}$$

It is important to avoid the stable zeroes of $T_2(s)$ appearing in $\boldsymbol{\theta}_1$ through (36), we need to get rid of all undetectable stable modes of $(D_{12}^{\perp}C_1, A_a)$, since the zeroes of $T_2(s)$ must be unobservable modes of $(D_{12}^{\perp}C_1, A_a)$. To this aim we can take a similarity transformation

$$T_a = \begin{bmatrix} R_a \\ N_a \end{bmatrix}, \quad T_a^{-1} = [\hat{R}_a \quad \hat{N}_a] \tag{46}$$

where $\hat{N}_a$ is the largest subspace satisfying

$$\begin{bmatrix} R_aA_a \\ D_{12}^{\perp}C_1 \end{bmatrix} \hat{N}_a = 0 \qquad \text{and } N_aA_a\hat{N}_a \text{ stable} \tag{47}$$

Thus we only need to solve the Riccati equation of lefted part after getting rid of $\hat{N}_a$

$$\begin{aligned} &(R_aA_a\hat{R}_a)X_a + X_a(R_aA_a\hat{R}_a)^{\top} + (X_a(D_{12}^{\perp}C_1\hat{R}_a)^{\top} + R_aB_1D_{11}^{\top}D_{12}^{\perp\top}) \\ &((D_{12}^{\perp}C_1\hat{R}_a)X_a + D_{12}^{\perp}D_{11}B_1^{\top}R_a^{\top}) + R_a((B_1 - B_2D_{12}^{+}D_{11})(B_1 - B_2D_{12}^{+}D_{11})^{\top} \\ &\quad - B_2D_{12}^{+}D_{12}^{+\top}B_2^{\top})R_a^{\top} = 0 \end{aligned} \tag{48}$$

and find a symmetric solution X_a of (48) such that

$$\hat{A}_a = R_aA_a\hat{R}_a - K_a^{\top}D_{12}^{\perp}C_1\hat{R}_a \tag{49}$$

is antistable, where

$$K_a = - D_{12}^{\perp}C_1\hat{R}_aX_a - D_{12}^{\perp}D_{11}\hat{B}_1^{\top}\hat{R}_a^{\top} \tag{50}$$

The normalized left annihilator $V_a(s)$ is

$$V_a = \{\hat{A}_a, \ -(K_a^{\top}D_{12}^{\perp} + R_aB_2D_{12}^{+}), \ D_{12}^{\perp}C_1\hat{R}_a, \ D_{12}^{\perp}\} \tag{51}$$

Similarly, to find the normalized right annihilator V_b (s), we define

$$A_b = A - B_1D_{21}^{+}C_2 \tag{52}$$

To get ride of all stable zeroes of $T_3(s)$, we can take a similarity transformation

$$T_b = [\hat{R}_b \quad \hat{N}_b], \quad T_b^{-1} = \begin{bmatrix} R_b \\ N_b \end{bmatrix} \tag{53}$$

where N_b is the largest subspace such that

$$N_b[A_b\hat{R}_b \quad B_1D_{21}^{\perp}] = 0 \qquad \text{and } N_bA_b\hat{N}_b \text{ stable} \tag{54}$$

Thus we only need to solve the Riccati equation of lefted part after getting rid of N_b,

$$\begin{aligned} &X_b(R_bA_b\hat{R}_b) + (R_bA_b\hat{R}_b)^{\top}X_b + (X_b(R_bB_1D_{21}^{\perp}) + \hat{R}_b^{\top}C_1^{\top}D_{11}D_{21}^{\perp}) \\ &((R_bB_1D_{21}^{\perp})^{\top}X_b + D_{21}^{\perp}D_{11}^{\top}C_1\hat{R}_b) + \hat{R}_b^{\top}((C_1 - D_{11}D_{21}^{+}C_2)^{\top}(C_1 - D_{11}D_{21}^{+}C_2) \\ &\qquad - C_2^{\top}D_{21}^{+\top}D_{21}^{+}C_2)\hat{R}_b = 0 \end{aligned} \tag{55}$$

and find a symmetric matrix X_b such that

$$\hat{A}_b = R_bA_b\hat{R}_b - R_bB_1D_{21}^{\perp}K_b \tag{56}$$

is antistable, where

$$K_b = (-X_bR_bB_1D_{21}^{\perp} - \hat{R}_b^{\top}C^{\top}D_{11}D_{21}^{\perp})^{\top} \tag{57}$$

The normalized right annihilator $V_b(s)$ is

$$V_b = \{\hat{A}_b,\ R_bB_1D_{21}^{\perp},\ -(D_{21}^{\perp}K_b + D_{21}^{+}C_2\hat{R}_b)),\ D_{21}^{\perp}\} \tag{58}$$

From the expressions of $V_a(s)$ and $V_b(s)$, we can write an explicit expression of Θ_0 by (34), and compute the explicit expression of Θ_1, a J-orthogonal complement of Θ_0. And then calculate the explicit expression of Π_a by (36), at last get L from (30) and (41). We can prove that the resulted Π_a is unimodular. All these computations are matrix calculations in state space intrinsically. We only write the last results, since the computation burden in huge and the space is limited. They can be summarized in theorem 2.

Theorem 2

The solvability condition of H_∞ regulation problem (18) of scaled plant $\overline{P}_\delta$ is

$$\begin{bmatrix} X_a & R_a\hat{R}_b \\ \hat{R}_b^{\top}R_a^{\top} & X_b \end{bmatrix} \geqslant 0 \tag{59}$$

The explicit expression of complete solution set for it's compersator is

$$C = (L_{11}\Phi + L_{12})(L_{21}\Phi + L_{22})^{-1}, \quad \Phi \in BH_{\infty}^{k\times r} \tag{60}$$

where L_{ij} are the four blocks of L in (41)

$$L = \{A_L,\ B_L,\ C_L,\ D_L\} \tag{61}$$

$$A_L = \begin{bmatrix} N_aA\hat{N}_a & -N(A_a\hat{R}_aX_a + G_1L_a^{\top} + G_2M_a^{\top}) \\ 0 & -\hat{R}_a^{\top}(A_a^{\top}R_a^{\top} - C_1^{\top}D_{12}^{\perp\top}K_a) \end{bmatrix} \tag{62.a}$$

$$B_L = \begin{bmatrix} N_a \\ 0 \end{bmatrix}[G_1\ \ G_2]D_c - \begin{bmatrix} 0 & N_a\hat{R}_b \\ I & 0 \end{bmatrix}\begin{bmatrix} X_a & R_a\hat{R}_b \\ \hat{R}_b^{\top}R_a^{\top} & Z_b \end{bmatrix}^{-1}\begin{bmatrix} L_a & -M_a \\ M_b & -L_b \end{bmatrix}D_c \tag{62.b}$$

$$C_L = \begin{bmatrix} -D_{21}^{+}C_1 \\ C_2 - D_{22}D_{12}^{+}C_1 \end{bmatrix}[\hat{N}_a\ \ -\hat{R}_aX_a] + \begin{bmatrix} D_{12}^{+} & -D_{12}^{+}D_{11} \\ D_{22}D_{12}^{+} & D_{21} - D_{22}D_{12}^{+}D_{11} \end{bmatrix}\begin{bmatrix} 0 & L_a^{\top} \\ 0 & M_a^{\top} \end{bmatrix} \tag{62.c}$$

$$D_L = \begin{bmatrix} D_{12}^{+} & -D_{12}^{+}D_{11} \\ D_{22}D_{12}^{+} & D_{21} - D_{22}D_{12}^{+}D_{11} \end{bmatrix}D_c \tag{62.d}$$

and

$$G_1 = B_2D_{12}^{+} \tag{63.a}$$

$$G_2 = B_1 - B_2 D_{12}^{+} D_{11} \tag{63.b}$$

$$L_a = K_a^{\top} D_{12}^{\perp} + R_a G_1 \tag{63.c}$$

$$M_a = - R_a B_1 + L_a D_{11} \tag{63.d}$$

$$L_b = K_b^{\top} D_{21}^{\perp\top} + \hat{R}_b^{\top} C_2^{\top} D_{21}^{+\top} \tag{63.e}$$

$$M_b = - \hat{R}_b^{\top} C_1^{\top} + L_b D_{11}^{\top} \tag{63.f}$$

$$D_0 = \begin{bmatrix} D_{12}^{\perp\top} & D_{11} D_{21}^{\top} \\ D_{11}^{\top} D_{12}^{\perp\top} & D_{21}^{\perp} \end{bmatrix} \tag{63.g}$$

4 Conclusion

In this paper we have solved the control problem for linear system with part uncertain parameters under bounded noise. The effect of parameters uncertainty can be converted to an equivalent effect of a new set of bounded noise. Thus above problem can be converted into a H_∞ control problem of a scaled difinite system under an extended bounded noise. Then we have dealt with the H_∞ standard problem of four blocks in general case ($D_{22} \neq 0$), and further obtained the explicit formulas of complete solution set for the parameter robust control problem. These formulas are related to the contolled plant parameters and design parameters directly, and can be programmed on computer conveniently.

In summary, this paper is concerned with parameter robust control problem which appears in many engineering design tasks, and gives the explicit formulas of complete solution set. These results will be with a good prospect in practical engineering applications.

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部分参数不确定线性系统的鲁棒控制

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摘　要　本文研究了部分参数不确定的线性系统在有界能量噪声作用下的控制问题，即参数鲁棒性控制问题。具有准确模型的飞机在不准确气动参数信息条件下的飞行控制就是一个典型例子。我们先把它简化为一个带正参数的 H_∞控制问题，然后采用 J 无损分解方法，推导了调节问题的可解性条件和动态反馈控制器的全部显式通解。

关键词：参数鲁棒性控制，H_∞标准控制问题，J 无损分解

分类号：TP13