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The Equivalent Forms On Chebyshev Pseudospectral Domain Decomposition For Elliptic Equations*

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Abstract This paper is devoted to establishing the Chebyshev pseudospectral domain decomposition scheme for elliptic equation, both one-dimensional and two-dimensional problems are discussed. The equivalent generalized variational forms for the Chebyshev pseudospectral domain decomposition scheme are given.

Key words Equivalent, Chebyshev pseudospectral method, domain decomposition, elliptic equation.

1 Introduction

In recent years, the parallel algorithm and the parallel computer provide the important tools for large-scale numerical computation. The domain decomposition method becomes more and more significant due to its easy parallel property. There have been a number of recent developments on the use of spectral techniques in more general geometries. The basic idea has been to partition the complete domain of the problem into severval subdomains.

The partitioning technique has been employed in finite-difference and finite-element methods. In the context of spectral methods, it dates from the late 1970s. Delves and Hall (1979) introduced a method which they called the global element method. Orszag (1980) decribed a technique for patching at interfaces. Morchoisne (1984) developed a method based on overlapping multiple domains. Patera (1984) used a variational formulation for what he termed the spectral-element method.

This paper is devoted to the Chebyshev pseudospectral domain decomposition method for elliptic equation, both one-dimensional and two-dimensional problems are considered, the patching technique is used. We find that the Chebyshev pseudospectral domain decomposition method is equivalent to a generalized variational method.

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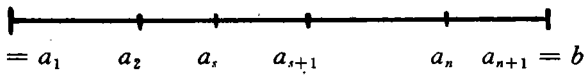
2 One-dimensional elliptic equation

Consider the Chebyshev pseudospectral domain decomposition method in I=(a,b) for the linear problem

$$\begin{cases} Lu \equiv -\upsilon u_{xx} + \lambda^2 u = f & x \in (a,b) \\ u(a) = u(b) = 0 \end{cases} \tag{2.1}$$

Where υ, λ are constants and $\upsilon > 0$.

We partition I into n subdomains as follows



Let

$$u_s^N(x) = \sum_{k=0}^{N_s} \tilde{u}_{s,k} T_k(\zeta) \qquad s = 1,\cdots,n$$

Where $x = \frac{a_{s+1}-a_s}{2}\zeta + \frac{a_{s+1}+a_s}{2}$, $\zeta \in [-1,1]$, and $T_k(\zeta)$ is the k-th Chebyshev polynomial.

The Chebyshev-Gauss-Lobatto points on $[a_s, b_s]$ are the following

$$x_j^{(s)} = \frac{a_{s+1}-a_s}{2}\cos\frac{\pi j}{N_s} + \frac{a_{s+1}+a_s}{2}, \qquad j = 0,\cdots,N_s$$

We construct the Chebyshev pseudospectral domain decomposition scheme by the following equations

$$\begin{cases} Lu_s^N - f|_{x=x_j^{(s)}} = 0 & j = 1,\cdots,N_s - 1; s = 1,\cdots,n \\ u_1^N(a_1) = 0, u_n^N(a_{n+1}) = 0; u_s^N(a_{s+1}) = u_{s+1}^N(a_{s+1}) \\ \frac{du_s^N}{dx}(a_{s+1}) = \frac{du_{s+1}^N}{dx}(a_{s+1}) \end{cases} \tag{2.2}$$

which the boundary conditions and the patching conditions or called the interface continuity conditions.

For any positive integer N_s, let P_{N_s} be the space of algebraic polynomials of degree at most N_s, set

$$P_{N_s}^0 = \{p \in P_{N_s} \text{ and } p(a_s) = p(a_{s+1}) = 0\}$$

Let $\tilde{I}_{N_s}: C^0(I_s) \to P_{N_s-2}$ defined by

$$(\tilde{I}_{N_s}\phi)(x_j^{(s)}) = \phi(x_j^{(s)}) \text{ for } 1 \le j \le N_s - 1$$

i.e., $\tilde{I}_{N_s}\phi$ is the interpolant of ϕ of degree $N_s - 2$ at the internal Gauss-Lobatto nodes.

By the definition of the interpolant operator, we know that the first equation of (2.2) is equivalent to the identities (between polynomials)

$$\tilde{I}_{N_s} Lu_s^N = \tilde{I}_{N_s} f \qquad s = 1,\cdots,n$$

or

$$-\upsilon u_{s,xx}^N + \lambda^2 \tilde{I}_{N_s} u_s^N = \tilde{I}_{N_s} f \qquad s = 1,\cdots,n \tag{2.3}$$

Setting

$$X_N = \{v \in C^0(I) \setminus \ v|I_s \in P_{N_s}, \text{for } s = 1,\cdots,n \text{ and } v = 0 \text{ on } x = a,b\}$$

For simplicity, we denote $v|_{I_s}$ by v_s. Multiplying each side of (2. 3) by $v_s\omega_s$ and integrating over I_s, we have

$$\int_{I_s}(-\upsilon u^N_{s,xx} + \lambda^2 \tilde{I}_{N_s} u^N_s)v_s\omega_s(x)\mathrm{d}x = \int_{I_s}(\tilde{I}_{N_s}f)v_s\omega_s(x)\mathrm{d}x \text{ for all } v \in X_N \quad (2.4)$$

By integrating by parts, we have from the boundary conditions and patching conditions that

$$\sum_{s=1}^{n}\int_{I_s}[\upsilon u^N_{s,x}(v_s\omega_s)_x + \lambda^2 \tilde{I}_{N_s} u^N_s v_s\omega_s(x)]\mathrm{d}x = \sum_{s=1}^{n}\int_{I_s}(\tilde{I}_{N_s}f)v_s\omega_s(x)\mathrm{d}x \text{ for all } v \in X_N \quad (2.5)$$

Conversely, let $u^N \in X_N$ be a solution of (2. 5). By integrating by parts, we get that

$$\sum_{s=1}^{n}\int_{I_s}[-\upsilon u^N_{s,xx} + \lambda^2 \tilde{I}_{N_s}(u^N_s - f)]v_s\omega_s(x)]\mathrm{d}x - \sum_{s=1}^{n-1}\upsilon \lim_{x\to a_{s+1}}(u^N_{s,x} - u^N_{s+1,x})(x)v(x)\omega_s(x)$$

$$= 0 \qquad \text{for all } v \in X_N$$

By choosing suitable v, we have that

$$\int_{I_s}[-\upsilon u^N_{s,xx} + \tilde{I}_{N_s}(\lambda^2 u^N_s - f)]v_s\omega_s(x)\mathrm{d}x = 0 \quad \forall\ v_s \in P_{N_s}, s = 1,\cdots,n$$

By the quadrature rule, we have for each $s=1,\cdots,n$ that

$$\int_{I_s}[-\upsilon u^N_{s,xx} + \tilde{I}_{N_s}(\lambda^2 u^N_s - f)]v_s\omega_s(x)\mathrm{d}x$$

$$\equiv \sum_{j=0}^{N_s}[-\upsilon u^N_{s,xx} + \tilde{I}_{N_s}(\lambda^2 u^N_s - f)](x^{(s)}_j)\omega_{s,j} = 0$$

where $\omega_{s,j}$ are the weights of Gauss-Lobatto quadrature formula. So we have

$$-\upsilon u^N_{s,xx} + \tilde{I}_{N_s}(\lambda^2 u^N_s - f)|_{x=x^{(s)}_j} = 0 \qquad s = 1,\cdots,n, j = 1,\cdots,N_s - 1$$

and

$$-\upsilon u^N_{s,xx} + \tilde{I}_{N_s}(\lambda^2 u^N_s - f) \equiv 0$$

Therefore

$$\sum_{s=1}^{n-1}\lim_{x\to a_{s+1}}(u^N_{s,x} - u^N_{s+1,x})(x)v_s(x)\omega_s(x) = 0 \qquad \forall\ v \in X_N$$

On the other hand, choosing suitable $v(a_{s+1}) \neq 0$, we deduce the boundary conditions and the patching conditions.

In fact, we have completed the proof of the following theorem.

Theorem 1. The Chebyshev pseudospectral domain decomposition scheme (2. 2) is equivalent to the generalized variational form (2. 5)

3 Two-dimensional elliptic equation

Consider the following two-dimensional elliptic equation

$$\begin{cases} L_u \equiv -\triangle u + \lambda^2 u = f & \text{in}\Omega \\ u|_{\partial\Omega} = 0 \end{cases} \tag{3.1}$$

where λ is a constant, and f is the known function, Ω is the region in Fig. 1.

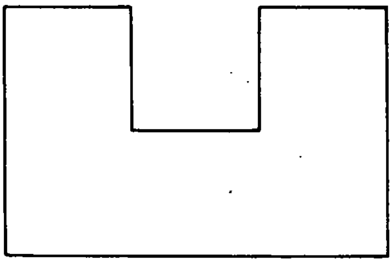


Fig. 1. Region Ω

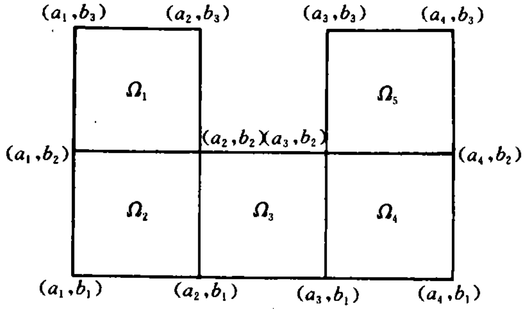


Fig. 2. The partition of Ω

We partition Ω into five subdomains $\Omega_s, s=1,\cdots,5$, and $\Omega=\bigcup_{s=1}^{5}\Omega_s$

Denote

$$\Gamma_s = \partial\Omega_s \cap \partial\Omega \qquad s = 1,\cdots,5$$

$$\Gamma_{s,s+1} = \partial\Omega_s \cap \partial\Omega_{s+1} \qquad s = 1,\cdots,4$$

The coordinates of the Chebyshev-Gauss-Radau points on Ω_s are denoted by $(x_m^{(s)}, y_n^{(s)})$, $m=0,\cdots,M_s, n=0,\cdots,N_s$, where M_s, N_s are positive integers. Assume that $M_1=M_2$, $M_4=M_5$, $N_1=N_5$, $N_2=N_3=N_4$. In fact

$$x_m^{(1)} = x_m^{(2)}, \qquad m = 0,\cdots,M_1$$

$$x_m^{(s)} = \frac{a_{s+1}-a_s}{2}\cos\frac{2\pi m}{2M_s+1} + \frac{a_{s+1}+a_s}{2}, \qquad s = 2,3,4; m = 0,\cdots,M_s$$

$$x_m^{(5)} = x_m^{(4)}, \qquad m = 0,\cdots,M_5$$

$$y_n^{(1)} = \frac{b_3-b_2}{2}\cos\frac{2\pi n}{2N_1+1} + \frac{b_3+b_2}{2}, \qquad n = 0,\cdots,N_1$$

$$y_n^{(2)} = \frac{b_2-b_1}{2}\cos\frac{2\pi n}{2N_2+1} + \frac{b_2+b_1}{2}, \qquad n = 0,\cdots,N_2$$

$$y_n^{(3)} = y_n^{(4)} = y_n^{(2)}, \qquad n = 0,\cdots,N_3$$

$$y_n^{(5)} = y_n^{(1)}, \qquad n = 0,\cdots,N_5$$

Let $(x,y)\in[a,b]\times[c,d]$, consider the following series form's solution

$$u_s^{M,N}(x,y) = \sum_{i=0}^{M_s}\sum_{j=0}^{N_s}\tilde{u}_{s,i,j}T_i(\zeta)T_j(\eta), \qquad s = 1,\cdots,5$$

where $x=\frac{b-a}{2}\zeta+\frac{b+a}{2}$, $y=\frac{d-c}{2}\eta+\frac{d+c}{2}$, ζ, $\eta\in[-1,1]$, and $T_i(\zeta)$, $T_j(\eta)$ are the i-th, j-th Chebyshev polynomials respectively.

The Chebyshev pseudospectral domain decomposition scheme of (3.1) is (3.2)-(3.

4).

$$Lu_s^{M,N} - f|_{x=x_m^{(s)},y=y_n^{(s)}} = 0, m = 1,\cdots,M_s, n = 1,\cdots,N_s, s = 1,\cdots,5 \quad (3.2)$$

Boundary conditions:

$$\begin{cases} u_1^{M,N}(a_1,y_n^{(1)}) = u_1^{M,N}(a_2,y_n^{(1)}) = u_1^{M,N}(x_m^{(1)},b_3) = 0, m = 0,\cdots,M_1, n = 0,\cdots,N_1 \\ u_2^{M,N}(a_1,y_n^{(2)}) = u_2^{M,N}(x_m^{(2)},b_1) = 0, m = 0,\cdots,M_2, n = 0,\cdots,N_2 \\ u_3^{M,N}(x_m^{(3)},b_1) = u_3^{M,N}(x_m^{(3)},b_2) = , m = 0,\cdots,M_3 \\ u_4^{M,N}(x_m^{(4)},b_1) = u_4^{M,N}(a_4,y_n^{(4)}) = 0, m = 0,\cdots,M_4, n = 0,\cdots,N_4 \\ u_5^{M,N}(a_3,y_n^{(5)}) = u_5^{M,N}(a_4,y_n^{(5)}) = u_5^{M,N}(x_m^{(5)},b_3) = 0, m = 0,\cdots,M_5, n = 0,\cdots,N_5 \end{cases} \quad (3.3)$$

Patching conditions:

$$\begin{cases} u_1^{M,N}(x_m^{(1)},b_2) = u_2^{M,N}(x_m^{(1)},b_2); \frac{\partial u_1^{M,N}}{\partial y}(x_m^{(1)},b_2) = \frac{\partial u_2^{M,N}}{\partial y}(x_m^{(1)},b_2), & m = 0,\cdots,M_1 \\ u_2^{M,N}(a_2,y_n^{(2)}) = u_3^{M,N}(a_2,y_n^{(2)}); \frac{\partial u_2^{M,N}}{\partial x}(a_2,y_n^{(2)}) = \frac{\partial u_3^{M,N}}{\partial x}(a_2,y_n^{(2)}), & n = 0,\cdots,N_2 \\ u_3^{M,N}(a_3,y_n^{(3)}) = u_4^{M,N}(a_3,y_n^{(3)}); \frac{\partial u_3^{M,N}}{\partial x}(a_3,y_n^{(3)}) = \frac{\partial u_4^{M,N}}{\partial x}(a_3,y_n^{(3)}), & n = 0,\cdots,N_3 \\ u_4^{M,N}(x_m^{(4)},b_2) = u_5^{M,N}(x_m^{(4)},b_2); \frac{\partial u_4^{M,N}}{\partial y}(x_m^{(4)},b_2) = \frac{\partial u_5^{M,N}}{\partial y}(x_m^{(4)},b_2), & m = 0,\cdots,M_4 \end{cases} \quad (3.4)$$

For simplicity of expression, we suppose that Ω_s is a rectangular region.

$\Omega_s=\{(x,y)|(x,y)\in[a,b]\times[c,d]\}$. Let $\omega(\zeta)=(1-\zeta^2)^{-\frac{1}{2}}$, $\zeta\in[-1,1]$. Define

$$\omega_{1,s}(x) = \omega(\zeta), x = \frac{b-a}{2}\zeta + \frac{b+a}{2}, \omega_{2,s}(y) = \omega(\zeta), y = \frac{d-c}{2}\zeta + \frac{d+c}{2}$$

For any positive integer M_s, N_s, let P_{M_s}, P_{N_s} be the spaces of algebraic polynomials of degree at most M_s, N_s respectively. Set

$$P_{M_s,N_s} = \{p(x,y): p(x,y) \in P_{M_s} \times P_{N_s}\}$$

$$P^0_{M_s,N_s} = \{p(x,y) \in P_{M_s,N_s} \text{ and } p(x,y)|_{\partial\Omega_s} = 0\}$$

Let $I_{M_s,N_s}: C^0(\Omega_s)\to P_{M_s-2,N_s-2}$ defined by

$$(I_{M_s,N_s}\Phi)(x_m^{(s)},y_n^{(s)}) = \Phi(x_m^{(s)},y_n^{(s)}), \text{for } m = 1,\cdots,M_s - 1; n = 1,\cdots,N_s - 1$$

i.e. I_{M_s,N_s} is the interpolant of Φ at the internal Chebyshev-Gauss-Radau nodes in Ω_s.

By the definition of I_{M_s,N_s}, we know that (3.2) is equivalent to the identities (between polynomials)

$$I_{M_s,N_s} Lu_s^{M,N} = I_{M_s,N_s} f; \ s = 1,\cdots,5 \quad (3.5)$$

or

$$-\frac{\partial^2}{\partial x^2}(I_{N_s}u_s^{M,N}) - \frac{\partial^2}{\partial y^2}(I_{M_s}u_s^{M,N}) + \lambda^2 I_{M_s,N_s}u_s^{M,N} = I_{M_s,N_s}f, s = 1,\cdots,5 \quad (3.6)$$

Setting

$$X_{M,N} = \{v \in C^0(\Omega) | v|_{\Omega_s} \in P_{M_s,N_s}, \text{for } s = 1,\cdots,5 \text{ and } v|_{\partial\Omega} = 0\}$$

For simplicity, we denote $v_s = v|_{\Omega_s}$. Let us multiply each side of (3.6) by $v_s\omega_{1,s}(x)\omega_{2,s}(y)$ and integrate over Ω_s. We have

$$\int_{\Omega_s}\left(-\frac{\partial^2}{\partial x^2}(I_{N_s}u_s^{M,N})-\frac{\partial^2}{\partial y^2}(I_{M_s}u_s^{M,N})+\lambda^2 I_{M_s,N_s}u_s^{M,N}\right)v_s\omega_{1,s}(x)\omega_{2,s}(y)\mathrm{d}x\mathrm{d}y$$
$$=\int_{\Omega_s}(I_{M_s,N_s}f)v_s\omega_{1,s}(x)\omega_{2,s}(y)\mathrm{d}x\mathrm{d}y,\text{for all } v\in X_{M,N} \tag{3.7}$$

By integrating by parts and summing for $s=1,\cdots,5$, noticing that $I_{M_1}=I_{M_2}, I_{N_2}=I_{N_3}=I_{N_4}, I_{M_4}=I_{M_5}$, we have from the boundary conditions and patching conditions that

$$\sum_{s=1}^{5}\int_{\Omega_s}[(I_{N_s}u_s^{M,N})_x(v_s\omega_{1,s}(x))_x\omega_{2,s}(y)+(I_{M_s}u_s^{M,N})_y(v_s\omega_{2,s}(y))_y\omega_{1,s}(x)$$
$$+\lambda^2 I_{M_s,N_s}u_s^{M,N}]\mathrm{d}x\mathrm{d}y \tag{3.8}$$
$$=\sum_{s=1}^{5}\int_{\Omega_s}(I_{M_s,N_s}f)v_s\omega_{1,s}(x)\omega_{2,s}(y)\mathrm{d}x\mathrm{d}y \quad \text{for all } v\in X_{M,N}$$

Conversely, suppose $u^{M,N}\in X_N$ be a solution of (4.4). By integrating by parts, we get that

$$\sum_{s=1}^{5}\int_{\Omega_s}[-\frac{\partial^2}{\partial x^2}(I_{N_s}u_s^{M,N})-\frac{\partial^2}{\partial y^2}(I_{M_s}u_s^{M,N})+\lambda^2 I_{M_s,N_s}(u_s^{M,N}-f)]v_s\omega_{1,s}(x)\omega_{2,s}(y)\mathrm{d}x\mathrm{d}y$$
$$-\int_{\Gamma_{1,2}}\left(\frac{\partial(I_{M_1}u_1^{M,N})}{\partial y}-\frac{\partial(I_{M_2}u_2^{M,N})}{\partial y}\right)v\omega_{1,1}(x)\omega_{2,1}(y)\mathrm{d}s$$
$$-\int_{\Gamma_{2,3}}\left(\frac{\partial(I_{N_2}u_2^{M,N})}{\partial x}-\frac{\partial(I_{N_3}u_3^{M,N})}{\partial x}\right)v\omega_{1,2}(x)\omega_{2,2}(y)\mathrm{d}s$$
$$-\int_{\Gamma_{3,4}}\left(\frac{\partial(I_{N_3}u_3^{M,N})}{\partial x}-\frac{\partial(I_{N_4}u_4^{M,N})}{\partial x}\right)v\omega_{1,3}(x)\omega_{2,3}(y)\mathrm{d}s$$
$$-\int_{\Gamma_{4,5}}\left(\frac{\partial(I_{M_4}u_4^{M,N})}{\partial y}-\frac{\partial(I_{M_5}u_5^{M,N})}{\partial y}\right)v\omega_{1,4}(x)\omega_{2,4}(y)\mathrm{d}s$$
$$=0 \qquad \text{for all } v\in X_{M,N} \tag{3.9}$$

By choosing suitable v, we have that

$$\int_{\Omega_s}[-\frac{\partial^2}{\partial x^2}(I_{N_s}u_s^{M,N})-\frac{\partial^2}{\partial y^2}(I_{M_s}u_s^{M,N})+\lambda^2 I_{M_s,N_s}(u_s^{M,N}-f)]v_s\omega_{1,s}(x)\omega_{2,s}(y)\mathrm{d}x\mathrm{d}y$$
$$=0 \qquad \forall\, v_s\in P^0_{M_s,N_s}, s=1,\cdots,5$$

By the quadrature rule, we have for each $s=1,\cdots,5$ that

$$\int_{\Omega_s}[-\frac{\partial^2}{\partial x^2}(I_{N_s}u_s^{M,N})-\frac{\partial^2}{\partial y^2}(I_{M_s}u_s^{M,N})+I_{M_s,N_s}(\lambda^2 u_s^{M,N}-f)]v_s\omega_{1,s}(x)\omega_{2,s}(y)\mathrm{d}x\mathrm{d}y$$
$$=\sum_{m=0}^{M_s}\sum_{n=0}^{N_s}[-\frac{\partial^2}{\partial x^2}(I_{N_s}u_s^{M,N})-\frac{\partial^2}{\partial y^2}(I_{M_s}u_s^{M,N})+I_{M_s,N_s}(\lambda^2 u_s^{M,N}-f)]v_s(x_m^{(s)},y_n^{(s)})\omega_{1,s}^m\omega_{2,s}^n$$
$$=\sum_{m=1}^{M_s}\sum_{n=1}^{N_s}[-\frac{\partial^2}{\partial x^2}(I_{N_s}u_s^{M,N})-\frac{\partial^2}{\partial y^2}(I_{M_s}u_s^{M,N})+I_{M_s,N_s}(\lambda^2 u_s^{M,N}-f)]v_s(x_m^{(s)},y_n^{(s)})\omega_{1,s}^m\omega_{2,s}^n$$

$= 0$

$$\forall\, v_s \in P^0_{M_s,N_s}$$

where $\omega_{1,s}^m, \omega_{2,s}^n$ are the weights of the Chebyshev-Gauss-Radau formula, thus

$$-\frac{\partial^2}{\partial x^2}(I_{N_s}u_s^{M,N}) - \frac{\partial^2}{\partial y^2}(I_{M_s}u_s^{M,N}) + I_{M_s,N_s}(\lambda^2 u_s^{M,N} - f)\mid_{x=x_m^{(s)},y_n^{(s)}} = 0$$

$$s = 1,\cdots,5; m = 1,\cdots,M_s; n = 1,\cdots,N_s.$$

Therefore $-\frac{\partial^2}{\partial x^2}(I_{N_s}u_s^{M,N}) - \frac{\partial^2}{\partial y^2}(I_{M_s}u_s^{M,N}) + I_{M_s,N_s}(\lambda^2 u_s^{M,N} - f) \equiv 0$ (3.10)

which is same to (3.5). On the other hand, we have from (3.9), (3.10) that

$$\int_{\Gamma_{1,2}} \left(\frac{\partial(I_{M_1}u_1^{M,N})}{\partial y} - \frac{\partial(I_{M_2}u_2^{M,N})}{\partial y}\right) v\omega_{1,1}(x)\omega_{2,1}(y)\mathrm{d}s$$
$$+ \int_{\Gamma_{2,3}} \left(\frac{\partial(I_{N_2}u_2^{M,N})}{\partial x} - \frac{\partial(I_{N_3}u_3^{M,N})}{\partial x}\right) v\omega_{1,2}(x)\omega_{2,2}(y)\mathrm{d}s$$
$$+ \int_{\Gamma_{3,4}} \left(\frac{\partial(I_{N_3}u_3^{M,N})}{\partial x} - \frac{\partial(I_{N_4}u_4^{M,N})}{\partial x}\right) v\omega_{1,3}(x)\omega_{2,3}(y)\mathrm{d}s$$
$$+ \int_{\Gamma_{4,5}} \left(\frac{\partial(I_{M_4}u_4^{M,N})}{\partial y} - \frac{\partial(I_{M_5}u_5^{M,N})}{\partial y}\right) v\omega_{1,4}(x)\omega_{2,4}(y)\mathrm{d}s$$
$$= 0 \quad \forall\, v \in X_{M,N}$$

By choosing suitable v, we can deduce the patching conditions (3.4), the boundary conditions is easily obtained due to $u^{M,N} \in X_{M,N}$.

In fact, we have completed the proof of the following theorem.

Theorem 2. The Chebyshev pseudospectral domain decomposition scheme (3.2)-(3.4) is equivalent to the generalized variational form (3.9).

4 Summary

This paper deals with the Chebyshev pseudospectral domain decomposition scheme for elliptic equation, both one-dimensional and two-dimensional problems are discussed. These new schemes can be paralleled in computation and easily applied to complex geometry. We also give the equivalent variational forms correspond to discrete Chebyshev pseudospectral domain decomposition scheme, which is significant to the analyses of the convergence and stability of the Chebyshev pseudospectral domain decomposition method.

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$e(t)$的真实数据及$\Delta e(t)=e(t)-\hat{e}(t)$。表1结果说明，当样本容量$m$取得充分大时、$\Delta e(t)\approx 0$，$\hat{e}(t)$可作为$e(t)$的"替代"数据。这正是定理6的结论。

表1

	t	1	10	15	20	25	30	35	40
m	$e(t)$	−1.974	−1.119	−1.042	−0.438	−0.800	−2.789	2.127	0.166
50	$\Delta e(t)$	−0.616	−0.354	−0.147	0.050	0.102	0.144	0.130	0.062
100	$\Delta e(t)$	−0.253	−0.206	−0.163	−0.043	−0.087	−0.055	−0.026	−0.028
300	$\Delta e(t)$	−0.072	−0.065	−0.057	−0.017	0.043	−0.037	−0.011	−0.026
500	$\Delta e(t)$	0.016	0.013	0.009	0.006	0.003	0.000	−0.002	−0.005

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椭圆型方程Chebyshev拟谱区域分解格式的等价形式

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摘　要　本文对一维、二维椭圆型方程建立了Chebyshev拟谱区域分解格式，对这种拟谱区域分解格式给出了一种等价的广义变分形式。

关键词　等价性，Chebyshev拟谱方法，区域分解，椭圆型方程。

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