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Universal Abstract Consistency Class and Universal Unifying Principle*

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Abstract Consistency is one of the most fundamental syntactic concepts in mathematical logic. By treating consistency in an abstract way, Smullyan presented abstract consistency class, and proved the socalled Smullyan's unifying principle. In this paper, considering various properties possessed by the class of consistent sets of wffs in first-order logic system, we generalize the concept of abstract consistency class into the most general form– universal abstract consistency class, and further prove its universal unifying principle. This result can be used to prove the completeness theorems of first-order logic system and the universal refutation method proposed by us.

Key words consistency, universal abstract consisency class, universal unifying principle

广义抽象协调类和广义合一原理

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摘　要　协调性是数理逻辑中最基本的语法概念之一。Smullyan 提出了抽象协调类概念, 并证明了相应的 Smullyan 合一原理。通过考察协调合式公式集类所具有的种种性质, 本文将抽象协调类概念推广至最一般的形式——广义抽象协调类, 并证明了相应的广义合一原理。这一结果可以用于证明一阶逻辑形式系统和我们所提出的广义反驳方法的完备性。

关键词　协调性, 广义抽象协调类, 广义合一原理
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The study of the interplay between syntax and semantics is fundamental to the study of logic systems. The syntactic concept – of *derivability* corresponds to the semantic concept = of *consequence*. As a syntactic counterpart of *satisfiability*, *consistency* is one of the most fundamental concepts in mathematical logic. To prove theorem:

Each consistent set Γ *of sentences has a model*

in first-order logic system, based on the consistency of Γ, we usually build our model (called the *canonical model*) out of syntactical materials[1,2]. By abstractly considering various properties possessed by the class of consistent sets of sentences, Smullyan proposed a concept of *abstract consistency class*, and proved the so-called Smullyan's unifying principle[3,4]. Smullyan's unifying principle is a generalization of the above theorem, which can yield a variety of important metatheorems. For example, the completeness theorems of the *first-order logic system*, the *semantic tableau method* or the $\mathscr{R}$ *refutation method* can be proved by means of Smullyan's unifying principle through constructing their abstract consistency classes respectively[5].

In this paper, by considering various properties possessed by the class of consistent sets of wffs in

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first-order logic system, we generalize the concept of the abstract consistency class into the most general form —— *universal abstract consistency class*, and further prove its *universal unifying principle*. This result can be used to prove the completeness theorems of first-order logic system and the *universal refutation method*[6] proposed by us.

1 Universal Abstract Consistency Class

First we introduce some terminologies and notations. The rest follows[5]. For each set X, let $\mathscr{P}(X)$ the power set of X, and $\# X$ the cardinality of X. N denotes the natural number set.

Lemma 1. If X be an infinite set, then there exist two subsets X', X'' of X such that $X = X' \cup X''$, $X' \cap X'' = \varnothing$ and $\# X' = \# X = \# X''$.

Let $\mathscr{F}$ be a first-order logic system. Its connectives are $\sim$ and $\vee$, its quantifier is $\forall$, while the rest connectives and quantifiers are just abbreviations. There are no variables except individual variables in $\mathscr{F}$. Thus all predicate and function symbols are constants. Assume that there are arbitrarily many function constants and predicate constants (at least one predicate constants) in $\mathscr{F}$. For each $n \in N$, $Funct(n)$ denotes the set of n-arity function constants in $\mathscr{F}$, and $Pred(n)$ the set of n-arity predicate constants in $\mathscr{F}$.

Σ is the alphabet of $\mathscr{F}$, and Var is the countably infinite set of individual variables of $\mathscr{F}$. $Term$ is the set of terms of $\mathscr{F}$. $\mathscr{L}(\mathscr{F})$ is the set of wffs of $\mathscr{F}$. $\Box$ means empty disjunction. Let $\widetilde{\mathscr{L}}(\mathscr{F}) = \mathscr{L}(\mathscr{F}) \cup \{\Box\}$. We regard $A \vee \Box$ and $\Box \vee A$ as abbreviations for A. Obviously, $\Box$ is unsatisfiable.

An interpretation $\mathscr{I} = <\mathscr{D}, \mathscr{I}_0>$ of $\mathscr{F}$ consists of a non-empty set $\mathscr{D}$ and a mapping $\mathscr{I}_0$. A function σ: $Var \rightarrow \mathscr{D}$ is called an assignment in $\mathscr{I}$. $\Sigma_{\mathscr{I}}$ denotes the set of assignments in $\mathscr{I}$. For each $t \in Term$ and $A \in \mathscr{L}(\mathscr{F})$, we use $\mathscr{I}(t)(\sigma)$ and $\mathscr{I}(A)(\sigma)$ to represent their semantic values respectively.

Definition 1. Assume that $\Gamma \subseteq \widetilde{\mathscr{L}}(\mathscr{F})$, $A \in \mathscr{L}(\mathscr{F})$ and $y_1, \cdots, y_k (k \geq 0)$ are all free variables which occur in $\sim \forall x A$. If $y_1, \cdots, y_k$ have no bound occurrences in A and there is a k-arity function constant g of $\mathscr{F}$ which does not occur in $\Gamma \cup \{A\}$, then $g(y_1, \cdots, y_k)$ is called a *Skolem term* of $\sim \forall x A$ *with respect to* Γ, where g is the corresponding *Skolem functor*.

Clearly, $g(y_1, \cdots, y_k)$ is free (substitutable) for x in A, hence $\vdash \sim \boldsymbol{S}^x_{g(y_1, \cdots, y_k)} A \supset \sim \forall x A$.

Definition 2. Assume that $A \in \widetilde{\mathscr{L}}(\mathscr{F})$, x is a bound variable of A and y does not occur in A. $\boldsymbol{K}^x_y A$ denotes the result gained by renaming designated bound occurrences of x with y in A.

Obviously, $\vdash \boldsymbol{K}^x_y A \equiv A$.

Definition 3. Assume that $\Gamma \subseteq \widetilde{\mathscr{L}}(\mathscr{F})$.

(1) We say that *there are enough function constants* in $\mathscr{F}$, iff for each $n \in \boldsymbol{N}$, $Funct(n)$ is infinite.

(2) We say that *there are the most function constants* in $\mathscr{F}$, iff for each $n \in \boldsymbol{N}$, $\# Funct(n) = \# Funct(0) \geq \# Var$ and $\# Funct(0) \geq \# Pred(n)$.

(3) We say that Γ is *sufficiently pure* in $\mathscr{F}$, iff for each $n \in \boldsymbol{N}$, there are $\# \mathscr{L}(\mathscr{F})$ n-arity function constants of $\mathscr{F}$ which do not occur in Γ.

Lemma 2. If there are the most function constants in $\mathscr{F}$ and $\Gamma \subseteq \widetilde{\mathscr{L}}(\mathscr{F})$ is a finite set, then Γ is sufficiently pure in $\mathscr{F}$.

Definition 4. Let $\mathscr{H} \subseteq \mathscr{P}(\widetilde{\mathscr{L}}(\mathscr{F}))$.

(1) $\mathscr{H}$ is *closed under subsets* iff when $\Gamma \in \mathscr{H}$ and $\Gamma' \subseteq \Gamma$, then $\Gamma' \in \mathscr{H}$.

(2) $\mathscr{H}$ is *of finite character* iff for each Γ, $\Gamma \in \mathscr{H}$ iff every finite subset of Γ is a member of $\mathscr{H}$.

Obviously, if $\mathscr{H} \subseteq \mathscr{P}(\widetilde{\mathscr{L}}(\mathscr{F}))$ is of finite character, then $\mathscr{H}$ is closed under subsets.

The concept of universal abstract consistency class is given as follows.

Definition 5. If $\mathscr{H} \subseteq \mathscr{P}(\widetilde{\mathscr{L}}(\mathscr{F}))$ satisfies:

(1) $\mathscr{H}$ is closed under subsets;

(2) Assume $\Gamma \in \mathscr{H}$ and $A, B \in \mathscr{L}(\mathscr{F})$, then

a) $\Box \notin$ F;

b) If A is an atomic formula, then $A \notin \Gamma$ or $\sim A \notin \Gamma$;

c) If $A \in \Gamma$, x is a bound variable of A and individual variable y does not occur in A, then $\Gamma \cup \{\boldsymbol{K}_y^x A\} \in \mathscr{H}$;

d) If $\sim\sim A \in \Gamma$, then $\Gamma \cup \{A\} \in \mathscr{H}$;

e) If $A \vee B \in \Gamma$, then $\Gamma \cup \{A\} \in \mathscr{H}$ or $\Gamma \cup \{B\} \in \mathscr{H}$;

f) If $\sim(A \vee B) \in \Gamma$ then $\Gamma \cup \{\sim A, \sim B\} \in \mathscr{H}$;

g) If $\forall xA \in \Gamma$ and term t is free for x in A, then $\Gamma \cup \{\boldsymbol{S}_t^x A\} \in \mathscr{H}$;

h) If $\sim\forall xA \in \Gamma$ and $g(y_1, \cdots, y_k)$ is a Skolem term of $\sim\forall xA$ with respect to Γ, then $\Gamma \cup \{\sim \boldsymbol{S}_{g(y_1,\cdots,y_k)}^x A\} \in \mathscr{H}$;

then we say that $\mathscr{H}$ is a *universal abstract consistency class* of $\mathscr{F}$.

According to Definition 5, we can conclude that

(1) Obviously, both $\approxeq$ and $\{\approxeq\}$ are universal abstract consistency classes of $\mathscr{F}$; neither $\mathscr{P}(\mathscr{L}(\mathscr{F}))$ nor $\mathscr{P}(\widetilde{\mathscr{L}}(\mathscr{F}))$ is a universal abstract consistency class of $\mathscr{F}$;

(2) If $\mathscr{H}$ is a universal abstract consistency class of $\mathscr{F}$, then $\approxeq \in \mathscr{H}$ and $\{\Box\} \notin \mathscr{H}$;

(3) If $\mathscr{H}$ is a universal abstract consistency class of $\mathscr{F}$ and $\Gamma \in \mathscr{H}$, then $\Box \notin \Gamma$.

Proposition 1. Let $\mathscr{H} = \{\Gamma \subseteq \widetilde{\mathscr{L}}(\mathscr{F}) \mid \Gamma$ is consistent$\}$, then $\mathscr{H}$ is a unviersal abstract consistency class of $\mathscr{F}$.

It is by means of various properties of $\mathscr{H}$ that we propose the universal abstract consistency class.

Lemma 3. Assume that $\mathscr{H}$ is a unviersal abstract consistency class of $\mathscr{F}$. Let

$\mathscr{H}' = \{\Gamma' \subseteq \widetilde{\mathscr{L}}(\mathscr{F}) \mid$ if $\Gamma \subseteq \Gamma'$ is a finite set, then $\Gamma \in \mathscr{H}\}$

Then $\mathscr{H} \subseteq \mathscr{H}'$ and $\mathscr{H}'$ is a unviersal abstract consistency class of finite character.

2 Universal Unifying Principle

In order to prove the universal unifying principle for the universal abstract consistency class, we introduce two important lemmas here.

Lemma 4. Assume that $\mathscr{H} \subseteq \mathscr{P}(\widetilde{\mathscr{L}}(\mathscr{F}))$ is a universal abstract consistency class of finite character and α is a limit ordinal. If $\{\Gamma_\xi \subseteq \widetilde{\mathscr{L}}(\mathscr{F}) \mid \xi < \alpha\} \subseteq \mathscr{H}$ satisfies:

$\Gamma_0 \subseteq \Gamma_1 \subseteq \cdots \subseteq \Gamma_\xi \subseteq \cdots, \quad \xi < \alpha$

then $\bigcup_{\xi<\alpha} \Gamma_\xi \in \mathscr{H}$

Proof. It is obvious since $\mathscr{H}$ is closed under subsets and of finite character. □

Lemma 5. Assume that $\mathscr{H}$ is a universal abstract consistency class of $\mathscr{F}$ which is of finite character and $\Gamma \in \mathscr{H}$. If $\Gamma \neq \approxeq$ is sufficiently pure in $\mathscr{F}$, then there is $\Gamma^* \in \mathscr{H}$ which satisfies:

1) $\Gamma \subseteq \Gamma^*$;

2) Γ^* is sufficiently pure in $\mathscr{F}$;

3) If $A \in \widetilde{\mathscr{L}}(\mathscr{F})$ and $\Gamma^* \cup \{A\} \in \mathscr{H}$, then $A \in \Gamma^*$.

Proof. See [7] for details. □

Theorem 1 (Universal Unifying Principle). Assume that $\mathscr{H}$ is a universal abstract consistency class of $\mathscr{F}$. If $\Gamma \in \mathscr{H}$ is sufficiently pure in $\mathscr{F}$, then Γ is satisfiable.

Proof. The theorem is trivial if Γ is empty, so we may assume $\Gamma \neq \approxeq$. $\mathscr{H}$ can be further supposed to be of finite character by Lemma 3. Since Γ is sufficiently pure in $\mathscr{F}$, it follows from Lemma 5 that there exists $\Gamma^* \in \mathscr{H}$ satisfying the conditions 1), 2) and 3).

Let $\mathscr{T} = < \mathscr{D}, \mathscr{T}_0 >$ and $\sigma \in \Sigma_{\mathscr{T}}$ defined as follows:

(1) Let $\mathscr{D} = Term$, i. e. Domain $\mathscr{D}$ is the term set of $\mathscr{F}$;

(2) If f is a n-arity function constant, then $\mathscr{F}_0(f): \mathscr{D} \to \mathscr{D}$ is defined as follows:

$$\mathscr{F}_0(f)(t_1, \cdots, t_n) = f(t_1, \cdots, t_n), \quad t_1, \cdots, t_n \in \mathscr{D}$$

If P is a n-arity predicate constant, then n-arity predicate $\mathscr{F}_0(P)$ on $\mathscr{D}$ is defined as follows:

$$\mathscr{F}_0(P)(t_1, \cdots, t_n) = true \quad \text{iff} \quad P(t_1, \cdots, t_n) \in \Gamma^*, \quad t_1, \cdots, t_n \in \mathscr{D}$$

(3) Assignment $\sigma: Var \to \mathscr{D}$ is defined by

$$\sigma(x) = x, \quad x \in Var$$

To prove $\mathscr{T}$ and σ satisfying Γ, we need only prove the following statement by structural induction:

$$\text{If } E \in \Gamma^*, \text{ then } \mathscr{K}(E)(\sigma) = true.$$

We consider the following cases:

(1) E is an atomic formula, obviously $\mathscr{K}(E)(\sigma) = true$.

(2) Suppose $E \in \Gamma^*$ and $E = \sim A$.

(a) A is an atomic formula. Since $\Gamma^* \in \mathscr{H}$ and $\sim A \in \Gamma^*$, we have $A \notin \Gamma^*$. Thus $\mathscr{K}(A)(\sigma) = false$, therefore $\mathscr{K}(E)(\sigma) = \mathscr{K}(\sim A)(\sigma) = true$.

(b) $A = \sim B$. Then $\sim\sim B \in \Gamma^*$, so $\Gamma^* \cup \{B\} \in \mathscr{H}$ so $B \in \Gamma^*$. Thus $\mathscr{K}(B)(\sigma) = true$ by inductive hypothesis, hence $\mathscr{K}(E)(\sigma) = true$.

(c) $A = B \vee C$. Then $\sim(B \vee C) \in \Gamma^*$, so $\Gamma^* \cup \{\sim B, \sim C\} \in \mathscr{H}$. *Thus* $\Gamma^* \cup \{\sim B\} \in \mathscr{H}$ *and* $\Gamma^* \cup \{\sim C\} \in \mathscr{H}$ *since* $\mathscr{H}$ *is closed under subsets, so* $\sim B \in \Gamma^*$ *and* $\sim C \in \Gamma^*$. *Hence* $\mathscr{K}(\sim B)(\sigma) = true = \mathscr{K}(\sim C)(\sigma)$ by inductive hypothesis, so $\mathscr{K}(\sim E)(\sigma) = \mathscr{K}(\sim(B \vee C))(\sigma) = true$. (d) $A = \forall xB$. Then $\sim \forall xB \in \Gamma^*$. Let $x_1, \cdots, x_m (m \geq 0)$ be all bound variables occurred in B. Since Var is countably infinite, then there exist m distinct individual variables $z_1, \cdots, z_m$ which do not occur in $\sim \forall xB$. We take each bound occurrence of $x_1, \cdots, x_m$ in B as designated bound occurrence, and let $\theta = \boldsymbol{K}^{x_1}_{z_1} \cdots \boldsymbol{K}^{x_m}_{z_m}$, then $\theta(\sim \forall xB) = \sim \forall x\theta(B)$ (note that even for some $i(1 \leq i \leq m)$ such that $x_i = x$, we do not rename the non-designated bound occurrences of x in the most outer level of $\sim \forall xB$). Thus $\Gamma^* \cup \{\sim \forall x\theta(B)\} \in \mathscr{H}$, so $\sim \forall x\theta(B) \in \Gamma^*$. Let $y_1, \cdots, y_k (k \geq 0)$ be all free variables occurred in $\sim \forall x\theta(B)$, then $y_1, \cdots, y_k, z_1, \cdots, z_m$ are distinct each other, and none of $y_1, \cdots, y_k$ has a bound occurrence in $\theta(B)$. Since Γ^* is sufficiently pure, then there is a k-arity function constant g which does not occur in $\Gamma^* \cup \{\theta(B)\}$. Hence $g(y_1, \cdots, y_k)$ is a Skolem term of $\sim \forall x\theta(B)$ with respect to Γ^*, then $\Gamma^* \cup \{\sim \boldsymbol{S}^x_{g(y_1, \cdots, y_k)}\theta(B)\} \in \mathscr{H}$, so $\sim \boldsymbol{S}^x_{g(y_1, \cdots, y_k)}\theta(B) \in \Gamma^*$. Therefore $\mathscr{K}(\sim \boldsymbol{S}^x_{g(y_1, \cdots, y_k)}\theta(B))(\sigma) = true$ by inductive hypothesis. Since $\mathscr{K}(g(y_1, \cdots, y_k))(\sigma) = g(y_1, \cdots, y_k)$ and $- B \equiv \theta(B)$, we have

$$\mathscr{K}(\sim B)(\sigma[x/g(y_1, \cdots, y_k)]) = \mathscr{K}(\sim\theta(B))(\sigma[x/\mathscr{K}(g(y_1, \cdots, y_k))(\sigma)]) = \mathscr{K}(\sim \boldsymbol{S}^x_{g(y_1, \cdots, y_k)}\theta(B))(\sigma) = true$$

Then $\mathscr{T}(B)(\sigma[x/g(y_1, \cdots, y_k)]) = false$, so $\mathscr{K}(\forall xB)(\sigma) = false$, so $\mathscr{K}(E)(\sigma) = true$.

(3) Suppose $E \in \Gamma^*$ and $E = A \vee B$.

Since $A \vee B \in \Gamma^*$, then $\Gamma^* \cup \{A\} \in \mathscr{H}$ or $\Gamma^* \cup \{B\} \in \mathscr{H}$. *Thus* $A \in \Gamma^*$ *or* $B \in \Gamma^*$, *hence* $\mathscr{K}(A)(\sigma) = true$ or $\mathscr{K}(B)(\sigma) = true$ by inductive hypothesis. Hence $\mathscr{K}(E)(\sigma) = \mathscr{K}(A \vee B)(\sigma) = true$. (4) Suppose $E \in \Gamma^*$ and $E = \forall xA$.

For any $t \in \mathscr{D}$, then $t \in Term$ and $\mathscr{K}(t)(\sigma) = t$. Let $x_1, \cdots, x_m (m \geq 0)$ be all bound variables occurred in A. Since Var is countably infinite, then there are m distince individual variables $y_1, \cdots, y_m$ which differ from x and do not occurred in A or t. We take every bound occurrence of $x_1, \cdots, x_m$ in A as designated bound occurrence, and let $\theta = \boldsymbol{K}^{x_1}_{y_1} \cdots \boldsymbol{K}^{x_m}_{y_m}$, then $\theta(\forall xA) = \forall x\theta(A)$ and t is free for x in $\theta(A)$. Since $\forall xA \in \Gamma^*$ and $\Gamma^* \in \mathscr{H}$, then $\Gamma^* \cup \{\forall x\theta(A)\} \in \mathscr{H}$, *so* $\forall x\theta(A) \in \Gamma^*$, *thus* $\boldsymbol{S}^x_t\theta(A) \in \Gamma^*$. By inductive hypothesis we have $\mathscr{K}(\boldsymbol{S}^x_t\theta(A))(\sigma) = true$. Since $- A \equiv \theta(A)$, then

$$\mathscr{K}(A)(\sigma[x/t]) = \mathscr{K}(\theta(A))(\sigma[x/\mathscr{K}(t)(\sigma)]) = \mathscr{K}(\boldsymbol{S}^x_t\theta(A))(\sigma) = true.$$

Because of the arbitrariness of t, we conclude that $\mathscr{K}(E)(\sigma) = \mathscr{K}(\forall xA)(\sigma) = true$. □

Corollary 1. Assume that there are most function constants in $\mathscr{F}$ and $\mathscr{H}$ is a universal abstract consistency class of $\mathscr{F}$. If $\Gamma \in \mathscr{H}$ is a finite set, then Γ is satisfiable.

Proof. It follows from Lemma 2 and Theorem 1 directly. □

It must be pointed out that

1) The purity condition for Γ in the universal unifying principle is really necessary, otherwise Γ may not be satisfiable. For example, suppose that there is only one individual constant c and one 3-arity predicate constant P in $\mathscr{F}$. Let $\Gamma = \{\sim \forall x(P(x,y,c) \vee \sim P(x,y,c))\}$ and $\mathscr{H} = \{\approxeq, \Gamma\}$. Clearly $\mathscr{H}$ is a universal abstract consistent class of $\mathscr{F}$. But $\Gamma \in \mathscr{H}$ is not sufficiently pure in $\mathscr{F}$ and Γ is unsatisfiable.

2) If $\mathscr{H}$ is a universal abstract consistency class of $\mathscr{F}$ and $\mathscr{F}'$ is an extension of $\mathscr{F}$ obtained by adding new constants to $\mathscr{F}$, then $\mathscr{H}$ may not be a universal abstract consistency class of $\mathscr{F}'$, and further there may not exist a universal abstract consistency class $\mathscr{H}'$ of $\mathscr{F}'$ such that $\mathscr{H} \subseteq \mathscr{H}'$. For example, it is the case when $\mathscr{F}'$ is an extension of $\mathscr{F}$ obtained by adding a new 1-arity function constant g.

3) For application of the universal unifying principle to a concrete problem, one can always assume that the purity condition holds for Γ. Otherwise, one can discuss the problem in an extension $\mathscr{F}'$ of $\mathscr{F}$. For example, to prove the following proposition:

Proposition 2. Each consistent subset Γ of $\mathscr{L}(\mathscr{F})$ is satisfiable.

we can obtain an extension $\mathscr{F}^*$ of $\mathscr{F}$ by adding $\# \mathscr{L}(\mathscr{F})$ n-arity new function constants for each $n \in N$. Then $\Gamma \subseteq \mathscr{L}(\mathscr{F}^*)$ is sufficiently pure in $\mathscr{F}^*$, and Γ is also consistent. Let

$\mathscr{H} = \{\Gamma' \subseteq \widetilde{\mathscr{L}}(\mathscr{F}^*) \mid \Gamma'$ is consistent$\}$

$\mathscr{H}$ is a universal abstract consistency class of $\mathscr{F}^*$ by Proposition 1, and $\Gamma \in \mathscr{H}$. Hence Γ is satisfiable by the universal unifying principle.

Proposition 2 is the first form of completeness theorem of first-order logic system. The second form of completeness theorem can follow from it directly:

Proposition 3. Assume that $\Gamma \subseteq \mathscr{L}(\mathscr{F})$ and $A \in \mathscr{L}(\mathscr{F})$. If $\Gamma \models A$, then $\Gamma \vdash A$.

3 Conclusion

Based on the concept of the abstract consistent class, we have proposed the concept of the universal abstract consistency class and proved its universal unifying principle. Universal unifying principle is a powerful logic tool for wide applications. By using it, the completeness theorems of the first-order logic system and the universal refutation method[6] proposed by us can by proved.

References

1 Shoenfield J R. Mathematical Logic. Addison-Wesley, 1967

2 Ebbinghaus H D, Flum J, Thomas W. Mathematical Logic.

3 Smullyan R M. A Unifying Principle in Quantification Theory. Proc. Nat. Acad. Sciences. U. S. A. 1963, 49: 828～832

4 Smullyan R M. First-order Logic. Springer-verlag, 1968

5 Andrews P B. An Introduction to Mathematical Logic and Type Theory: To Truth Through Proof. Academic Press, Inc. 1986

6 Wang Bingshan, Li Zhoujun, Chen Huowang. Universal Refutation Method and Its Soundness and Completeness. Technical Report, NUDT, 1995

7 Wang Bingshan, Li Zhoujun. Universal Abstract Consistency Class and Universal Unifying Principle. Technical Report. NUDT, 1995