

## 多准则下反导作战单、多遭遇点预测模型\*

李龙跃,刘付显,杨国哲,王东旭,王菊  
(空军工程大学防空反导学院,陕西西安 710051)

**摘要:**整个反导射击过程可以简单看作一个基于预测的遭遇点起始的,不断重复、修正的过程。分析遭遇点预测的时间和空间前提,并给出遭遇点预测的3个基本准则;在多准则下建立了单遭遇点预测模型,给出了模型公式中未知量的转化方法;分射击-观察-射击和射击-射击两种情况建立了多准则下多遭遇点预测和优化模型,并给出了拦截弹最晚发射时间的计算方法;就观察时机对遭遇点预测的影响进行了分析与建模。部分研究属于探索性的研究,相关结论对指控模型开发和实施连续反导,从方法和作战理念方面提供了一些参考。

**关键词:**反导作战;单遭遇点;多遭遇点;观察时机;预测模型

**中图分类号:**TJO **文献标志码:**A **文章编号:**1001-2486(2015)04-179-09

## Multi-criteria forecast models of antimissile single/multiple impact points

LI Longyue, LIU Fuxian, YANG Guozhe, WANG Dongxu, WANG Ju

(Air and Missile Defense College, Air Force Engineering University, Xi'an 710051, China)

**Abstract:**The whole antimissile process can be viewed as an iteratively process based on impact points forecasting which shows that impact points was mainly focused on by present research in the following way. The time-space requirements for impact points forecasting were analyzed, and 3 forecasting criterions were proposed. The antimissile single impact points forecasting model at 3 criterions was established, and the unknown variables of the model were changed into values which radar can measure. For shoot-look-shoot scenario and shoot-shoot scenario, antimissile multiple impact points forecasting and optimization model was gave at 3 criterions, and the calculating method of latest launch time was discussed. The impact of different look occasion was analyzed and modeled during impact points forecasting process. Partial theories and methods was exploratory research, related conclusions made may be benefits for antimissile sustained shooting decision making.

**Key words:** antimissile battle; single impact point; multiple impact points; look occasion; forecast model

防空作战中,预判或评估杀伤目标的一个重要办法就是概率判断法,而当前和未来大多反导武器发射的拦截弹进入到与目标弹头的交战空域时,通过弹上综合制导雷达上传的信息进行目标捕获和跟踪,最终释放拦截器依靠动能直接撞击杀伤目标,属于典型的直接碰撞杀伤(Hit To Kill, HTK)。一些文献围绕反导遭遇点预测相关问题进行了前期研究,荆武兴等<sup>[1]</sup>考虑了不同发射场景下,如何通过快速规划拦截弹飞行方案和发射时间窗口来提高反导拦截弹的反应能力,重点研究反导遭遇时间的计算方法;王君等<sup>[2]</sup>研究了地空导弹(非机动)空气动力目标过程中遭遇点的预测问题;张友安等<sup>[3]</sup>在论文中提到了舰空导弹一般情况下弹目遭遇点的计算方法;万雨君等<sup>[4]</sup>

建立了基于“当前”统计情况的遭遇点预测模型,而其重点是给出对机动目标的拦截弹导引律。国外方面,美国、以色列研制和部署的反导系统最为先进,相关研究也有很多成果,主要集中于反导拦截器的导航与制导控制等方面<sup>[5-8]</sup>。

除了上述研究,对于反导遭遇点预测还有一些问题需要关注:一是要判断火力单元从空间和时间上是否适宜对目标进行拦截;二是要考虑不同的预测准则,如果反导武器的射击准则不同,那么遭遇点的预测结果也就不同;三是面对同一波次多个目标攻击,除了单遭遇点计算,还要考虑计算多个遭遇点和拦截弹的发射时间间隔等问题。李龙跃等从以上三个问题出发,分析遭遇点预测的空间-时间前提,建立多准则下的单个、多个遭

\* 收稿日期:2014-10-24

基金项目:全军军事学研究生资助项目(2014JY525)

作者简介:李龙跃(1988—),男,河南驻马店人,博士研究生,E-mail:lilong\_yue@126.com;

刘付显(通信作者),男,教授,博士,博士生导师,E-mail:liuxqh@126.com

遇点预测模型,并就一些模型细节进行了探讨。

### 1 遭遇点预测的前提和预测准则

#### 1.1 遭遇点预测的空间-时间前提

拦截飞机目标时通常要求目标对火力单元的航路捷径小于火力单元的最大杀伤航路捷径,且目标高度处在火力单元的杀伤高度范围内,飞机在不做大角度机动时,满足上述两点情况下一般都通过火力单元杀伤区<sup>[9]</sup>。同样,当弹道导弹目标飞经火力单元水平杀伤范围时,此时目标高度和相对火力单元的斜距分别处于该火力单元导弹拦截高度、导弹拦截斜距范围内时,目标才有机会被拦截。因此,目标弹道与火力单元杀伤区必须要有交点,用  $T$  表示目标,  $R_{max}, R_{min}$  分别表示火力单元最大和最小拦截斜距、 $H_{max}, H_{min}$  分别表示最大和最小拦截高度,则空间拦截可行性条件为  $R_{max} \leq R_T \leq R_{min}, H_{max} \leq H_T \leq H_{min}$ 。

这里把目标离开杀伤区时刻与到达杀伤区时刻间的时间段称为杀伤时间窗口,当目标在时刻  $t$  (介于到达杀伤区时刻和离开杀伤区时刻之间),拦截器飞至预测遭遇点,则满足拦截可行性时间条件。杀伤时间窗口可根据目标当前时刻,位置,速度以及火力单元位置,拦截弹杀伤最大、最小拦截斜距,最大、最小拦截高度迭代计算得到。理论上,只要目标在某时刻  $t$  尚未穿越火力单元杀伤区近界都可能是遭遇点,图 1 是遭遇点可预测性判断流程图。

#### 1.2 遭遇点预测准则

根据反导射击习惯,将遭遇点预测准则分为:尽远准则、尽近准则和最大概率拦截准则。1) 尽远准则。对弹道导弹目标一般情况下要尽量实现尽早发现、尽远拦截,尽远准则是为了充分利用拦截时间,尽可能多地增加拦截次数,提高总的拦截概率<sup>[10]</sup>。2) 尽近准则。为了先拦截更重要的目标,有时把相对次要的目标尽可能排在最后面拦截,满足在杀伤区内构成一次射击条件即可。3) 最大概率拦截准则。武器系统雷达对目标探测跟踪的时间越长,对目标的轨道参数预测越准确,计算出的遭遇点误差也越小。最大概率拦截准则要求遭遇点处于拦截器对目标的最佳杀伤点处,此时拦截概率最大。图 2 是 3 种准则下的遭遇点空间位置示意图。

### 2 多准则下单遭遇点预测模型

表 1 是本文所用参数及说明。

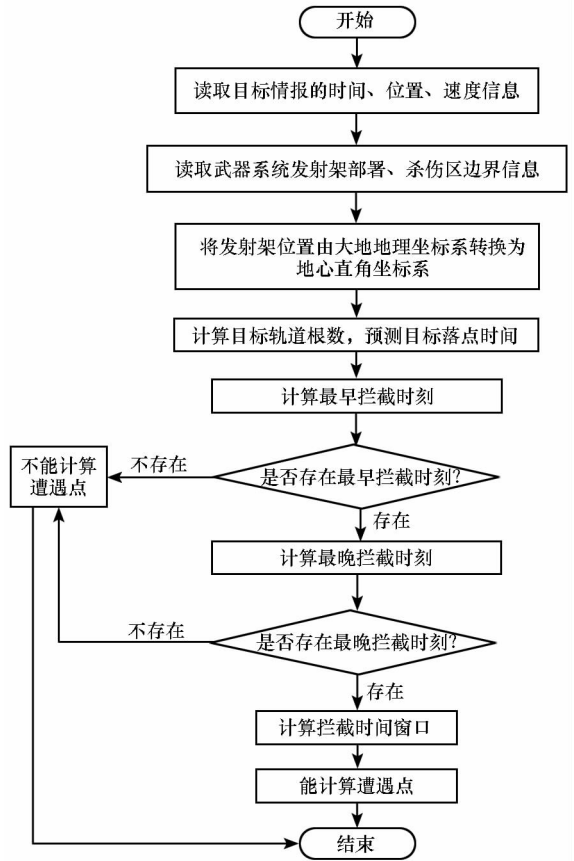


图 1 遭遇点可预测性判断流程图

Fig. 1 Forecastability estimating process of impact point

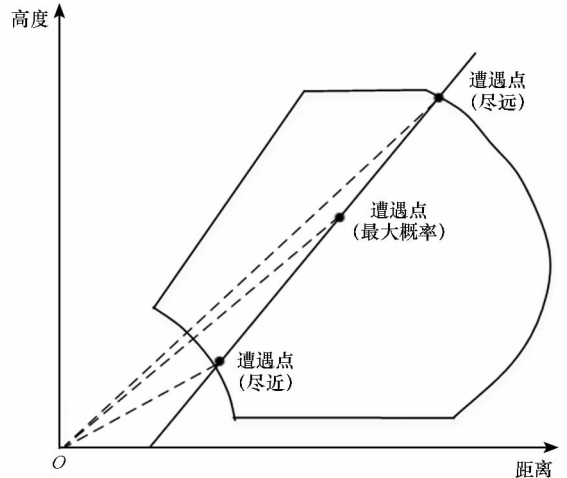


图 2 不同准则下的遭遇点空间位置示意图

Fig. 2 Schematic of impact points at different criteria

#### 2.1 基本计算公式

单遭遇点拦截,是对来袭目标只进行一次拦截,一般情况下是由拦截需求和拦截边界条件限制决定的,一般适用于两种情况:一是只需要一次拦截就能满足拦截目标的需要;二是由于边界条件的限制,仅够完成一次拦截所需要的资源或时间。过程中可能出现两种情况:1) 同时到达的多

个目标<sup>[11-12]</sup>,研究相对成熟;2)目标到达具有一定时间间隔,研究相对较少。以尽远准则为例,尽远准则下遭遇点即为目标杀伤区远界与目标弹道的交点位置。根据相对运动原理,目标飞到最近遭遇点的时间与拦截弹从发射点到达遭遇点的时间相等,即

$$t_{pq(\text{hit})} - t_{pq(\text{launch})} = t_{pjy(\text{hit})} - t_{pjy(\text{launch})} \quad (1)$$

以 $\angle \vec{LI} \vec{OX}$ ,  $\angle \vec{LI} \vec{OY}$ ,  $\angle \vec{LI} \vec{OZ}$ 分别表示目标弹道与三个坐标轴的夹角,由式(1)中时间的参考不变性<sup>[13]</sup>可得

$$\begin{aligned} t_{pq(\text{hit})} - t_{pq(\text{launch})} &= t_{pjy(\text{hit})} - t_{pjy(\text{launch})} \\ &= \frac{x_{pjy(\text{hit})} - x_{pjy(\text{launch})}}{v_o \sin\alpha \sin\beta \text{sign}(\cos \angle \vec{LI} \vec{OX})} \\ &= \frac{y_{pjy(\text{hit})} - y_{pjy(\text{launch})}}{v_o \sin\alpha \cos\beta \text{sign}(\cos \angle \vec{LI} \vec{OY})} \\ &= \frac{z_{pjy(\text{hit})} - z_{pjy(\text{launch})}}{v_o \cos\alpha \text{sign}(\cos \angle \vec{LI} \vec{OZ})} \end{aligned} \quad (2)$$

表1 参数及说明

Tab. 1 Parameters and description

参数	说明
$L, D_p$	目标落点和发射架位置
$q$	弹道导弹目标编号标识
$p$	武器系统发射架编号标识
$v_m, v_o$	拦截弹和目标平均速度
$\alpha, \delta, \varepsilon, \phi, \beta$	目标的再入角、航路角、方位角、俯仰角和水平面的投影与y轴的夹角
$t_{p(\text{hit})}q$	p对q弹目遭遇时刻点
$t_{pqi}$	p对q第i次拦截过程所消耗时间
$t_{p(\text{launch})}q$	弹目遭遇时刻点对应的拦截弹发射时刻点
$t_1$	拦截弹允许的最晚发射时间
jj, zd, jj	尽远、尽近和最大拦截概率3个准则的标识
$\Delta t_{pq(\text{launch})}$	拦截弹发射时间间隔
$(x(\cdot), y(\cdot), z(\cdot))$	$t(\cdot)$ 时刻目标位置坐标
$L$	目标所处空间点
$L_i$	第i个遭遇点编号标识
$(x^i(\cdot), y^i(\cdot), z^i(\cdot))$	$t(\cdot)$ 时刻,第i个遭遇点空间位置坐标
$P(x^i(\cdot), y^i(\cdot), z^i(\cdot))$	$t(\cdot)$ 时刻,第i个遭遇点拦截弹对目标的杀伤概率
$S_{pi}$	目标当前位置与发射架pi之间距离

$$\text{sign}(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases} \quad (3)$$

由式(2)可计算出尽远准则下单遭遇点的坐标

$$\begin{cases} x_{pjy(\text{hit})}q \\ y_{pjy(\text{hit})}q \\ z_{pjy(\text{hit})}q \end{cases} = \begin{cases} x_{pjy(\text{launch})}q \\ y_{pjy(\text{launch})}q + (t_{pjy(\text{hit})}q - t_{pjy(\text{launch})}q) v_o \sin\alpha \sin\beta \text{sign}(\cos \angle \vec{LI} \vec{OX}) \\ z_{pjy(\text{launch})}q \end{cases} \quad (4)$$

$$t_{pjy(\text{launch})}q v_o \begin{cases} \sin\alpha \sin\beta \text{sign}(\cos \angle \vec{LI} \vec{OX}) \\ \sin\alpha \cos\beta \text{sign}(\cos \angle \vec{LI} \vec{OY}) \\ \cos\alpha \text{sign}(\cos \angle \vec{LI} \vec{OZ}) \end{cases}$$

尽近准则、最大概率拦截准则与尽远准则下的单遭遇点计算一样,变量取不同值即可得到相应的不同准则下的公式,可用统一的式(5)来计算。

$$\begin{cases} x_{p(\text{hit})}q \\ y_{p(\text{hit})}q \\ z_{p(\text{hit})}q \end{cases} = \begin{cases} x_{p(\text{launch})}q \\ y_{p(\text{launch})}q + (t_{p(\text{hit})}q - t_{p(\text{launch})}q) v_o \sin\alpha \sin\beta \text{sign}(\cos \angle \vec{LI} \vec{OX}) \\ z_{p(\text{launch})}q \end{cases} \quad (5)$$

$$t_{p(\text{launch})}q v_o \begin{cases} \sin\alpha \sin\beta \text{sign}(\cos \angle \vec{LI} \vec{OX}) \\ \sin\alpha \cos\beta \text{sign}(\cos \angle \vec{LI} \vec{OY}) \\ \cos\alpha \text{sign}(\cos \angle \vec{LI} \vec{OZ}) \end{cases}$$

## 2.2 基本计算公式中未知量的转化

2.1节中的计算公式中并非所有未知量都是实际可获取的,仍以尽远准则下单遭遇点预测为例,假设目标位置L点相对于发射架pi的测量值为 $(S_{pi}, \phi_{pi}, \varepsilon_{pi})$ ,则

$$\begin{cases} x_{pjy(\text{launch})}q = S_{pi} \cdot \cos\phi_{pi} \cdot \cos\varepsilon_{pi} \\ y_{pjy(\text{launch})}q = S_{pi} \cdot \cos\phi_{pi} \cdot \sin\varepsilon_{pi} \\ z_{pjy(\text{launch})}q = S_{pi} \cdot \sin\phi_{pi} \end{cases} \quad (6)$$

现在已经对式(4)中第一项完成了转化, $\alpha, \beta, LI$ 分别可由式(7)~(9)计算得到。

$$\alpha = \arccos\left(\frac{S_{pi} \sin\phi_{pi}}{LI}\right) \quad (7)$$

$$\beta = \arcsin\left(\frac{S_{pi} \cos\phi_{pi} \cos\varepsilon_{pi} - x_L}{LI \arccos\alpha}\right) \quad (8)$$

$$LI = \sqrt{(x_{pjy(\text{launch})}q - x_L)^2 + (y_{pjy(\text{launch})}q - y_L)^2 + (z_{pjy(\text{launch})}q - z_L)^2} \quad (9)$$

由式(1)、式(2)可得

$$t_{pq1} = t_{pjy(\text{hit})}q - t_{pjy(\text{launch})}q = \frac{LL_1}{v_o} = \frac{D_{pi} L_1}{v_m} \quad (10)$$

其中

$$(LL_1)^2 = (x_{pjy(\text{launch})}q - x_{pjy(\text{hit})}q)^2 + (y_{pjy(\text{launch})}q - y_{pjy(\text{hit})}q)^2 + (z_{pjy(\text{launch})}q - z_{pjy(\text{hit})}q)^2 \quad (11)$$

$$D_{pi}L_i = \sqrt{(x_{pi} - x_{pjy(hit)q})^2 + (y_{pi} - y_{pjy(hit)q})^2 + (z_{pjy(launch)q})^2} \tag{12}$$

先将式(9)代入式(7)、式(8),式(11)、式(12)代入式(10),最后再将式(8)、式(10)、式(7)代入式(4)可得未知量的转化后的尽远准则下单遭遇点计算模型。 $t_{pq1}$ 计算方法见 3.1 节,至此尽远准则下单遭遇点预测模型仅需输入目标  $L$  相对于发射架  $pi$  的测量值、目标落点坐标和发射架  $pi$  的坐标就可计算出遭遇点坐标,最大概率拦截准则和尽近准则下单遭遇点预测方法与上述方法一致。

### 3 射击-观察-射击模式下多准则多遭遇点预测模型

#### 3.1 尽远准则下多遭遇点预测模型

假设“观察”时间忽略不计,尽远准则下第  $i$  个遭遇点坐标为  $(x_{pjy(hit)q}^i, y_{pjy(hit)q}^i, z_{pjy(hit)q}^i)$ , 已知

$$t_{pq1} = \frac{-2v_o [(x_{pi} - x_{pjy(launch)q}^i) \sin\alpha \sin\beta - (y_{pi} - y_{pjy(launch)q}^i) \sin\alpha \sin\beta + z_{pjy(launch)q}^i]}{2(v_o^2 - v_m^2)} \pm \frac{\sqrt{b^2 - 4(v_o^2 - v_m^2)(S_{pi}^2 + S_L^2 - 2x_{pi}x_{pjy(launch)q}^i + 2y_{pi}y_{pjy(launch)q}^i)}}{2(v_o^2 - v_m^2)} \tag{15}$$

显然式(15)有 2 个解,取  $t_{pq1}$  值较小的解,同样方法可求  $t_{pq^i}$ ,将其代入式(12),可以计算出尽远准则下第  $i$  个发射架发射的拦截弹第  $i$  次拦截目标时的遭遇点坐标( $i$  为非 0 正整数)。

$$\begin{cases} x_{pjy(hit)q}^i = S_{pi} \left\{ \begin{matrix} \cos\phi_{pi} \cos\varepsilon_{pi} \\ \cos\phi_{pi} \sin\varepsilon_{pi} + \sum_{i=1}^i t_{pq^i} v_o \\ \sin\phi_{pi} \end{matrix} \right\} \cdot \begin{cases} \text{sign}(\cos \angle \vec{LI} \vec{OX}) \\ \text{sign}(\cos \angle \vec{LI} \vec{OY}) \\ \text{sign}(\cos \angle \vec{LI} \vec{OZ}) \end{cases} \\ \left. \begin{matrix} \sin\left(\arccos \frac{S_{pi} \sin\phi_{pi}}{LI}\right) \sin\left(\arcsin \frac{S_{pi} \cos\phi_{pi} \cos\varepsilon_{pi} - x_L}{L \sin\left(\arccos \frac{S_{pi} \sin\phi_{pi}}{LI}\right)}\right) \\ \sin\left(\arccos \frac{S_{pi} \sin\phi_{pi}}{LI}\right) \cos\left(\arcsin \frac{S_{pi} \cos\phi_{pi} \cos\varepsilon_{pi} - x_L}{L \sin\left(\arccos \frac{S_{pi} \sin\phi_{pi}}{LI}\right)}\right) \\ \cos\left(\arccos \frac{S_{pi} \sin\phi_{pi}}{LI}\right) \end{matrix} \right\} \end{cases} \tag{16}$$

#### 3.2 尽近准则下最晚发射时间点计算模型

令  $t_{pq1}^i$  表示发射架  $D_{pi}$  发射的拦截弹到达  $L_i$  的最晚时间,则

发射架  $p$  对目标  $q$  第  $i$  次拦截的时间为  $t_{pq^i}$ ,目标位置  $L$  点相对于发射架  $pi$  的测量值为  $(S_{pi}, \phi_{pi}, \varepsilon_{pi})$ ,预测多遭遇点的一个重要依据仍是相对运动时间相等原则,参照前面单遭遇点计算方法。式(10)中  $t_{pq1}$  是未知量,将式(7)、式(8)代入式(9),再将式(9)代入式(10)得

$$(v_o^2 - v_m^2)t_{pq1}^2 + 2v_o [(x_{pi} - x_{pjy(launch)q}^i) \sin\alpha \sin\beta - (y_{pi} - y_{pjy(launch)q}^i) \sin\alpha \sin\beta + z_{pjy(launch)q}^i] t_{pq1} + S_{pi}^2 + S_L^2 - 2x_{pi}x_{pjy(launch)q}^i + 2y_{pi}y_{pjy(launch)q}^i = 0 \tag{13}$$

式(13)中除了  $t_{pq1}$ ,其他参数均为已知。可看出式(13)是个以  $t_{pq1}$  为变量的一元二次方程,依据一元二次方程求根公式

$$t_{pq1} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{14}$$

其中  $a, b, c$  分别对应的是式(13)中的二次项系数,一次项系数和常数项,则  $t_{pq1}$  为

$$t_{pq^i}^j = \frac{\sqrt{(x_{pj} - x_{pjy(hit)q}^i)^2 + (y_{pj} - y_{pjy(hit)q}^i)^2 + z_{pj}^2}}{v_m} \tag{17}$$

尽近准则下最晚发射时间点对应的弹头位置本质上是尽近遭遇点的上一个遭遇点,则

$$\begin{cases} x_{pjy(hit)q}^{i-1} = x_L - A \sin\alpha \sin\beta \text{sign}(\cos \angle \vec{LI} \vec{OX}) \\ y_{pjy(hit)q}^{i-1} = y_L - A \sin\alpha \cos\beta \text{sign}(\cos \angle \vec{LI} \vec{OY}) \\ z_{pjy(hit)q}^{i-1} = -A \cos\alpha \text{sign}(\cos \angle \vec{LI} \vec{OZ}) \\ A = |IL_i| + v_o t_{pq1}^i \\ |IL_i| = \sqrt{(x_L - x_{pjy(hit)q}^i)^2 + (y_L - y_{pjy(hit)q}^i)^2 + (z_{pjy(hit)q}^i)^2} \end{cases} \tag{18}$$

$$t_{pq^{(i-1)}}^j = \frac{\sqrt{(x_{pj} - x_{pjy(hit)q}^{i-1})^2 + (y_{pj} - y_{pjy(hit)q}^{i-1})^2 + z_{pjy(hit)q}^{i-1}}}{v_m} \tag{19}$$

同理可计算第 1 枚拦截弹发射时间点对应的目标坐标和允许的最晚发射时间  $t_{1o}$ 。

$$\begin{cases} x_{pjj(\text{hit})q}^1 = x_l - B\sin\alpha\sin\beta\text{sign}(\cos\angle \vec{Ll} \vec{OX}) \\ y_{pjj(\text{hit})q}^1 = y_l - B\sin\alpha\cos\beta\text{sign}(\cos\angle \vec{Ll} \vec{OY}) \\ z_{pjj(\text{hit})q}^1 = -B\cos\alpha\text{sign}(\cos\angle \vec{Ll} \vec{OZ}) \\ B = |Ll_i| + v_o \sum_i^n t_{pqi}^i \\ t_l = t_{pjj(\text{launch})q} + \\ \sqrt{(x_{pjj(\text{launch})q} - x_{pjj(\text{hit})q}^1)^2 + (y_{pjj(\text{launch})q} - y_{pjj(\text{hit})q}^1)^2 + (z_{pjj(\text{launch})q} - z_{pjj(\text{hit})q}^1)^2} \\ v_m \end{cases} \quad (20)$$

### 3.3 最大概率拦截准则下遭遇点预测模型

目标在杀伤区中心附近被杀伤概率最大,因此最大概率拦截可以看作是在尽远准则的基础上滞后一段时间发射,设滞后发射时间为  $\Delta t$ ,则  $0 \leq \Delta t \leq t_1 - t_{pjj(\text{launch})q}$ ,滞后  $\Delta t$  发射拦截弹时目标的位置为  $L'$ 。最大概率拦截准则下遭遇点预测与“滞后发射时间  $\Delta t$ ”有关,假设已知每个遭遇点对目标的杀伤概率  $P(x_{pzd(\text{hit})q}^i, y_{pzd(\text{hit})q}^i, z_{pzd(\text{hit})q}^i)$ ,则最大概率拦截准则下的多遭遇点优化模型为

$$\begin{aligned} \max P &= 1 - \prod_{i=1}^n [1 - P(x_{pzd(\text{hit})q}^i, y_{pzd(\text{hit})q}^i, z_{pzd(\text{hit})q}^i, \Delta t)] \\ \text{s. t.} &\begin{cases} x_{pjj(\text{hit})q} \geq x_{pzd(\text{hit})q}^1 > \dots > x_{pzd(\text{hit})q}^i \geq x_{pjj(\text{hit})q} \\ y_{pjj(\text{hit})q} \leq y_{pzd(\text{hit})q}^1 < \dots < y_{pzd(\text{hit})q}^i \leq y_{pjj(\text{hit})q} \\ z_{pjj(\text{hit})q} \geq z_{pzd(\text{hit})q}^1 > \dots > z_{pzd(\text{hit})q}^i \geq z_{pjj(\text{hit})q} \\ 0 \leq \Delta t \leq t_1 - t_{pjj(\text{launch})q} \end{cases} \end{aligned} \quad (21)$$

与尽远准则下遭遇点计算思路非常类似,式(22)是预测遭遇点  $L_i$  的坐标计算公式,注意到如果将式(22)方形框中项变为  $\Delta t$ ,则式(22)变为点  $L'$  的坐标计算公式,如果将式(22)方形框中项变为  $(\Delta t + t_{pqi})$ ,则式(22)变为点  $L_1$  的坐标计算公式。

$$\begin{cases} x_{pzd(\text{hit})q}^i \\ y_{pzd(\text{hit})q}^i \\ z_{pzd(\text{hit})q}^i \end{cases} = \begin{cases} x_{pjj(\text{launch})q} \\ y_{pjj(\text{launch})q} \\ z_{pjj(\text{launch})q} \end{cases} + \boxed{(\Delta t + \sum_{i=1}^i t_{pqi})} \cdot$$

$$\begin{cases} \sin\alpha\sin\beta\text{sign}(\cos\angle \vec{LL'} \vec{OX}) \\ \sin\alpha\cos\beta\text{sign}(\cos\angle \vec{LL'} \vec{OY}) \\ \cos\alpha\text{sign}(\cos\angle \vec{LL'} \vec{OZ}) \end{cases} \quad (22)$$

将式(22)代入到式(21)中,便可进行相关模型求解工作。

## 4 射击-射击模式下多准则多遭遇点预测模型

### 4.1 尽远准则下多遭遇点预测模型

在射击-射击模式下需要考虑拦截弹发射时间间隔的确定问题,发射时间间隔分固定、可变两种。设发射时间间隔为  $\Delta t_{pq(\text{launch})}$ ,  $(\Delta t_{pq(\text{launch})})_{\min}$  为最小发射间隔,设分配给目标的拦截弹数量为  $Q$ ,其余参数沿用表1中给定参数,则

$$(\Delta t_{pq(\text{launch})})_{\max} = \frac{t_1 - t_{pjj(\text{launch})q}}{Q - 1}$$

$$(\Delta t_{pq(\text{launch})})_{\min} \leq \Delta t_{pq(\text{launch})} \leq (\Delta t_{pq(\text{launch})})_{\max}$$

记  $\Delta t_{pq(\text{launch})}^i$  为本次发射与下次发射之间的时间间隔,则

$$(\Delta t_{pq(\text{launch})}^i)_{\min} = (\Delta t_{pq(\text{launch})})_{\min}$$

$$(\Delta t_{pq(\text{launch})}^i)_{\max} = (t_1 - t_{pjj(\text{launch})q}) - (Q - 2) \cdot (\Delta t_{pq(\text{launch})})_{\min}$$

$$(\Delta t_{pq(\text{launch})})_{\min} \leq \Delta t_{pq(\text{launch})}^i \leq (\Delta t_{pq(\text{launch})})_{\max}$$

$(t_1 - t_{pjj(\text{launch})q})$  实质上是武器系统对目标的可发射时间。基于固定时间间隔  $\Delta t_{pq(\text{launch})}$  和可变时间间隔  $\Delta t_{pq(\text{launch})}^i$  的取值范围,类比第3节,发射-发射模式下尽远准则遭遇点计算公式为

$$\begin{cases} x_{pjj(\text{hit})q}^i \\ y_{pjj(\text{hit})q}^i \\ z_{pjj(\text{hit})q}^i \end{cases} = \begin{cases} x_{pjj(\text{launch})q} \\ y_{pjj(\text{launch})q} \\ z_{pjj(\text{launch})q} \end{cases} + \left( \sum_{i=1}^{i-1} t_{pq(\text{launch})}^i + t_{pqi} \right) v_o \cdot$$

$$\begin{cases} \sin\alpha\sin\beta\text{sign}(\cos\angle \vec{LL} \vec{OX}) \\ \sin\alpha\cos\beta\text{sign}(\cos\angle \vec{LL} \vec{OY}) \\ \cos\alpha\text{sign}(\cos\angle \vec{LL} \vec{OZ}) \end{cases} \quad (23)$$

$$t_{pqi} = \frac{\sqrt{(x_{pi} - x_{pjj(\text{hit})q}^i)^2 + (y_{pjj(\text{hit})q}^i)^2 + (z_{pjj(\text{hit})q}^i)^2}}{v_m} \quad (24)$$

### 4.2 最大拦截概率准则下多遭遇点预测模型

射击-射击模式下最大概率拦截准则时多遭遇点优化模型可将式(21)进行改写,分固定发射

间隔和可变发射间隔两种情况。

固定发射间隔可由式(25)求得。

$$\begin{aligned} \max P &= 1 - \prod_{i=1}^n [1 - P(x_{\text{pzd}}^i(\text{hit})_q, y_{\text{pzd}}^i(\text{hit})_q, \\ &\quad z_{\text{pzd}}^i(\text{hit})_q, \Delta t, \Delta t_{\text{pq}}(\text{launch}))] \\ \text{s. t. } &\begin{cases} x_{\text{pjy}}(\text{hit})_q \geq x_{\text{pzd}}^1(\text{hit})_q > x_{\text{pzd}}^2(\text{hit})_q > \dots > x_{\text{pzd}}^i(\text{hit})_q \geq x_{\text{pjj}}(\text{hit})_q \\ y_{\text{pjy}}(\text{hit})_q \leq y_{\text{pzd}}^1(\text{hit})_q < y_{\text{pzd}}^2(\text{hit})_q < \dots < y_{\text{pzd}}^i(\text{hit})_q \leq y_{\text{pjj}}(\text{hit})_q \\ z_{\text{pjy}}(\text{hit})_q \geq z_{\text{pzd}}^1(\text{hit})_q > z_{\text{pzd}}^2(\text{hit})_q > \dots > z_{\text{pzd}}^i(\text{hit})_q \geq z_{\text{pjj}}(\text{hit})_q \\ 0 \leq \Delta t \leq t_1 - n\Delta t_{\text{pq}}(\text{launch}) \\ (\Delta t_{\text{pq}}(\text{launch}))_{\min} \leq \Delta t_{\text{pq}}(\text{launch}) \leq \frac{t_1 - t_{\text{pjy}}(\text{launch})_q}{Q-1} \end{cases} \end{aligned} \quad (25)$$

可变发射间隔可由式(26)求得。

$$\begin{aligned} \max P &= 1 - \prod_{i=1}^n [1 - P(x_{\text{pzd}}^i(\text{hit})_q, y_{\text{pzd}}^i(\text{hit})_q, \\ &\quad z_{\text{pzd}}^i(\text{hit})_q, \Delta t, \Delta t_{\text{pq}}^i(\text{launch}))] \\ \text{s. t. } &\begin{cases} x_{\text{pjy}}(\text{hit})_q \geq x_{\text{pzd}}^1(\text{hit})_q > x_{\text{pzd}}^2(\text{hit})_q > \dots > x_{\text{pzd}}^i(\text{hit})_q \geq x_{\text{pjj}}(\text{hit})_q \\ y_{\text{pjy}}(\text{hit})_q \leq y_{\text{pzd}}^1(\text{hit})_q < y_{\text{pzd}}^2(\text{hit})_q < \dots < y_{\text{pzd}}^i(\text{hit})_q \leq y_{\text{pjj}}(\text{hit})_q \\ z_{\text{pjy}}(\text{hit})_q \geq z_{\text{pzd}}^1(\text{hit})_q > z_{\text{pzd}}^2(\text{hit})_q > \dots > z_{\text{pzd}}^i(\text{hit})_q \geq z_{\text{pjj}}(\text{hit})_q \\ 0 \leq \Delta t \leq t_1 - \sum_{i=1}^{n-1} \Delta t_{\text{pq}}^i(\text{launch}) \\ (\Delta t_{\text{pq}}(\text{launch}))_{\min} \leq \Delta t_{\text{pq}}^i(\text{launch}) \leq [(t_1 - t_{\text{pjy}}(\text{launch})_q) - \\ (Q-2)(\Delta t_{\text{pq}}(\text{launch}))_{\min}] \end{cases} \end{aligned} \quad (26)$$

式(25)、式(26)中的遭遇点计算公式为

$$\begin{cases} x_{\text{pzd}}^i(\text{hit})_q \\ y_{\text{pzd}}^i(\text{hit})_q \\ z_{\text{pzd}}^i(\text{hit})_q \end{cases} = \begin{cases} x_{\text{pjy}}^i(\text{launch})_q \\ y_{\text{pjy}}^i(\text{launch})_q + (iv_o \Delta t_{\text{pq}}(\text{launch}) + \Delta t + t_{\text{pqi}}) \cdot \\ z_{\text{pjy}}^i(\text{launch})_q \end{cases}$$

$$v_o \begin{cases} \sin\alpha \sin\beta \text{sign}(\cos \angle \overrightarrow{LL'} \overrightarrow{OX}) \\ \sin\alpha \cos\beta \text{sign}(\cos \angle \overrightarrow{LL'} \overrightarrow{OY}) \\ \cos\alpha \text{sign}(\cos \angle \overrightarrow{LL'} \overrightarrow{OZ}) \end{cases} \quad (27)$$

$$\begin{cases} x_{\text{pzd}}^i(\text{hit})_q \\ y_{\text{pzd}}^i(\text{hit})_q \\ z_{\text{pzd}}^i(\text{hit})_q \end{cases} = \begin{cases} x_{\text{pjy}}^i(\text{launch})_q \\ y_{\text{pjy}}^i(\text{launch})_q + (\sum_{i=1}^{i-1} \Delta t_{\text{pq}}^i(\text{launch}) + \Delta t + t_{\text{pqi}}) \cdot \\ z_{\text{pjy}}^i(\text{launch})_q \end{cases}$$

$$v_o \begin{cases} \sin\alpha \sin\beta \text{sign}(\cos \angle \overrightarrow{LL'} \overrightarrow{OX}) \\ \sin\alpha \cos\beta \text{sign}(\cos \angle \overrightarrow{LL'} \overrightarrow{OY}) \\ \cos\alpha \text{sign}(\cos \angle \overrightarrow{LL'} \overrightarrow{OZ}) \end{cases} \quad (28)$$

$$t_{\text{pqi}} = \frac{\sqrt{(x_{\text{pi}} - x_{\text{pzd}}^i(\text{hit})_q)^2 + (y_{\text{pzd}}^i(\text{hit})_q)^2 + (z_{\text{pzd}}^i(\text{hit})_q)^2}}{v_m} \quad (29)$$

如果把式(28)中的推迟发射时间  $\Delta t$  看成首次发射时间间隔,即  $\Delta t_{\text{pq}}^0(\text{launch}) = \Delta t$ ,有

$$\sum_{i=1}^{i-1} \Delta t_{\text{pq}}^i(\text{launch}) + \Delta t = \sum_{i=0}^{i-1} \Delta t_{\text{pq}}^i(\text{launch}) \quad (30)$$

据此可对式(29)做进一步化简。

### 5 观察时机对遭遇点预测影响分析

由于存在杀伤效果观察环节,射击-观察-射击模式要比射击-射击模式更节省拦截弹资源,除了以上两种模式外,还有典型如“射击-观察-观察-射击-射击”“射击-射击-观察-射击”“射击-观察-射击-射击”等混合模式,实际作战也可以从“射击-射击-观察-射击”“射击-观察-射击-射击”中择优选取。观察时机的选取对遭遇点预测影响很大,尽远准则下的遭遇点预测需要尽远、尽快完成对目标的拦截过程。

假设尽远准则下拦截作战消耗时间为  $t_{\text{total}}$ ,把其中第  $i$  个遭遇点  $(x_{\text{pjy}}^i(\text{hit})_q, y_{\text{pjy}}^i(\text{hit})_q, z_{\text{pjy}}^i(\text{hit})_q)$  定为观察时机(设之前的遭遇点有  $a$  个),该点即为下一次遭遇点预测的起点,同样可以计算出后续遭遇点(设为  $b$  个)的坐标,已知  $\Delta t_{\text{pq}}^i(\text{launch})$  为拦截弹本次发射与下次发射之间的时间间隔,其余参数与表 1 设置一致,则

$$\begin{aligned} \min t_{\text{total}} &= \sum_{i=1}^{a-1} \Delta t_{\text{pq}}^i(\text{launch}) + \sum_{i=a+1}^{a+b-1} \Delta t_{\text{pq}}^i(\text{launch}) + t_{\text{pqa}} + t_{\text{pq}(a+b)} \\ \text{s. t. } &\begin{cases} (\Delta t_{\text{pq}}(\text{launch}))_{\min} \leq \Delta t_{\text{pq}}^i(\text{launch}) \leq \frac{|D_{\text{pi}} L_i|}{v_m} \\ a + b = n \\ (\sum_{i=1}^{a-1} \Delta t_{\text{pq}}^i(\text{launch}) + \sum_{i=a+1}^{a+b-1} \Delta t_{\text{pq}}^i(\text{launch}) + t_{\text{pqa}} + t_{\text{pq}(a+b)}) \\ \leq (t_1 - t_{\text{pjy}}(\text{launch})_q) \end{cases} \end{aligned} \quad (31)$$

式(31)中的  $(x_{\text{pjy}}^a(\text{hit})_q, y_{\text{pjy}}^a(\text{hit})_q, z_{\text{pjy}}^a(\text{hit})_q)$  和  $t_{\text{pqa}}$  计算公式为

$$\begin{cases} x_{\text{pjy}}^a(\text{hit})_q \\ y_{\text{pjy}}^a(\text{hit})_q \\ z_{\text{pjy}}^a(\text{hit})_q \end{cases} = \begin{cases} x_{\text{pjy}}^a(\text{launch})_q \\ y_{\text{pjy}}^a(\text{launch})_q + (\sum_{i=1}^{a-1} \Delta t_{\text{pq}}^i(\text{launch}) + t_{\text{pqa}}) v_o \cdot \\ z_{\text{pjy}}^a(\text{launch})_q \end{cases}$$

$$v_o \begin{cases} \sin\alpha \sin\beta \text{sign}(\cos \angle \overrightarrow{LI} \overrightarrow{OX}) \\ \sin\alpha \cos\beta \text{sign}(\cos \angle \overrightarrow{LI} \overrightarrow{OY}) \\ \cos\alpha \text{sign}(\cos \angle \overrightarrow{LI} \overrightarrow{OZ}) \end{cases} \quad (32)$$

$$t_{\text{pqa}} = \frac{\sqrt{(x_{\text{pi}} - x_{\text{pjy}}^a(\text{hit})_q)^2 + (y_{\text{pjy}}^a(\text{hit})_q)^2 + (z_{\text{pjy}}^a(\text{hit})_q)^2}}{v_m} \quad (33)$$

式(31)中的  $(x_{\text{pjy}}^{a+b}(\text{hit})_q, y_{\text{pjy}}^{a+b}(\text{hit})_q, z_{\text{pjy}}^{a+b}(\text{hit})_q)$  和  $t_{\text{pq}(a+b)}$  计算公式为

$$\begin{cases} x_{\text{pjy}}^{a+b}(\text{hit})_q \\ y_{\text{pjy}}^{a+b}(\text{hit})_q \\ z_{\text{pjy}}^{a+b}(\text{hit})_q \end{cases} = \begin{cases} x_{\text{pjy}}^a(\text{hit})_q \\ y_{\text{pjy}}^a(\text{hit})_q + (\sum_{i=a+1}^{a+b-1} \Delta t_{\text{pq}}^i(\text{launch}) + t_{\text{pq}(a+b)}) \cdot \\ z_{\text{pjy}}^a(\text{hit})_q \end{cases}$$

$$v_o \begin{cases} \sin\alpha\sin\beta\text{sign}(\cos\angle \vec{LI} \vec{OX}) \\ \sin\alpha\cos\beta\text{sign}(\cos\angle \vec{LI} \vec{OY}) \\ \cos\alpha\text{sign}(\cos\angle \vec{LI} \vec{OZ}) \end{cases} \quad (34)$$

$$t_{pq(a+b)} = \frac{\sqrt{(x_{pi} - x_{pjy(\text{hit})q}^{a+b})^2 + (y_{pjy(\text{hit})q}^{a+b})^2 + (z_{pjy(\text{hit})q}^{a+b})^2}}{v_m} \quad (35)$$

类式(27)和式(28),最大概率准则下观察时机选取不同时遭遇点优化模型见式(36)。由式(30)知  $\Delta t_{pq(\text{launch})q}^0 = \Delta t$ ,同理可设  $\Delta t_{aq(\text{launch})}^a = t_{pqa}$ 表示第  $a$  次射击的间隔,可对式(36)进行进一步化简。

$$\begin{aligned} \max P &= 1 - \prod_{i=1}^n [1 - P(x_{pzd(\text{hit})q}^i, y_{pzd(\text{hit})q}^i, \\ &\quad z_{pzd(\text{hit})q}^i, \Delta t, \Delta t_{pq(\text{launch})}^i)] \\ \text{s. t. } &\begin{cases} x_{pjy(\text{hit})q} \geq x_{pzd(\text{hit})q}^1 > x_{pzd(\text{hit})q}^2 > \dots > x_{pzd(\text{hit})q}^i \geq x_{pjj(\text{hit})q} \\ y_{pjy(\text{hit})q} \leq y_{pzd(\text{hit})q}^1 < y_{pzd(\text{hit})q}^2 < \dots < y_{pzd(\text{hit})q}^i \leq y_{pjj(\text{hit})q} \\ z_{pjy(\text{hit})q} \geq z_{pzd(\text{hit})q}^1 > z_{pzd(\text{hit})q}^2 > \dots > z_{pzd(\text{hit})q}^i \geq z_{pjj(\text{hit})q} \\ 0 \leq \Delta t \leq t_1 - \sum_{i=1}^{n-1} \Delta t_{pq(\text{launch})}^i, i \neq a \\ (\Delta t_{pq(\text{launch})})_{\min} \leq \Delta t_{pq(\text{launch})}^i \leq \\ [(t_1 - t_{pjy(\text{launch})q}) - (Q-3)(\Delta t_{pq(\text{launch})})_{\min} - t_{pqa}] \\ \sum_{i=1}^{a+b-1} \Delta t_{pq(\text{launch})}^i + \Delta t + t_{pqa} + t_{pq(a+b)} \leq (t_1 - t_{pjy(\text{launch})q}) \end{cases} \end{aligned} \quad (36)$$

## 6 实例分析

假设有2枚来袭弹道导弹,见表2,已知2个目标的弹道与武器杀伤区远、近界的交点<sup>[14]</sup>,设杀伤区边缘的杀伤概率为0.6。

显然,根据表2不同准则下的单遭遇点坐标可以直接得到。假设目标1只能有一个遭遇点,对目标2分别采取射击-观察-射击和射击-射击两种模式,对目标3采取射击-射击-观察-射击模式,则尽远准则下3个目标的遭遇点坐标和杀伤概率见表3。

限于篇幅,不再算出其他准则下3个目标的遭遇点坐标和杀伤概率。根据目标2的参数,代入射击-观察-射击模式下的最大概率拦截遭遇点计算模型,可得滞后发射时间  $\Delta t$  与杀伤概率之间的关系图,如图3所示。在拦截目标2的过程中,  $\Delta t$  不能大于30s,否则不能保证两次拦截,两个遭遇点的总杀伤概率在  $\Delta t = 30\text{s}$  时取得最大值,不再写出具体结果。对目标2,代入射击-射击模式下的最大概率拦截遭遇点计算模型,这里取发射间隔为10s和20s,经过仿真得到滞后发射时间与每个遭遇点及总杀伤概率之间的关系,见图4和图5。

表2 相关已知参数<sup>[14]</sup>

Tab.2 Known parameters of antimissile scenario

	目标1	目标2	目标3
杀伤区与弹道交点(远界)	(207.746 4,0,74.952 9)	(152.081 1,0,142.665 3)	(174.201 7,0,126.408 5)
杀伤区与弹道交点(近界)	(175.478 6,0,42.665 3)	(94.346 1,0,42.665 3)	(90.478 6,0, 42.557 7)
杀伤纵深中心点	(186.622 4,0,58.809 1)	(123.213 6,0,92.665 3)	(132.340 1,0, 84.536 9)
杀伤纵深	39.149 2	93.487 6	111.904 2
最大杀伤率	0.7	0.9	0.8

表3 尽远准则下3个目标的遭遇点坐标和杀伤概率

Tab.3 Impact points coordinates and kill probabilities at the farthest-criterion

	第1个遭遇点	第2个遭遇点	第3个遭遇点	杀伤概率
目标1	(207.746 4,0,74.952 9)	—	—	0.600 0
目标2(射击-观察-射击)	(152.081 1,0,142.665 3)	(102.151 6,0,86.184 9)	—	0.769 0
目标2(射击-射击)	(152.081 1,0,142.665 3)	(160.052 0,0,125.183 3)	(131.881 2,0,107.678 0)	0.873 3
目标3	(174.201 7,0,126.408 5)	(160.052 0,0, 112.238 7)	(96.294 6,0,48.481 3)	0.867 3

图4和图5可以看出目标2在发射-发射模式下,发射间隔为10s时,最佳滞后发射时间为45s;发射间隔为20s时,最佳滞后发射时间为36s

(前提是不小于武器系统最小发射间隔)。本文模型除了算出遭遇点坐标外,还可以计算出最佳滞后发射时间,见表4。

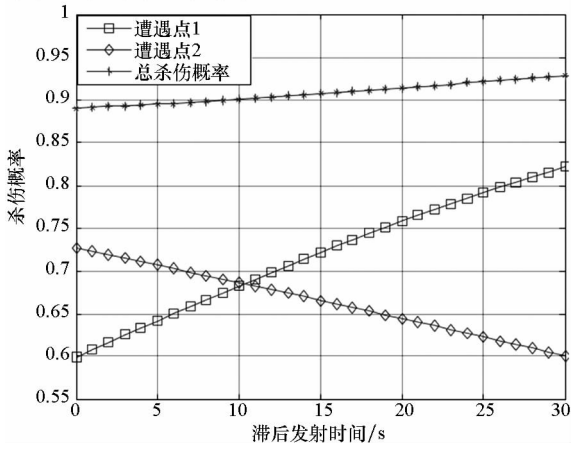


图 3 对目标 2 在射击-观察-射击模式下  
滞后发射时间与杀伤概率关系图

Fig. 3 Relationship between interceptors' delayed launch time and kill probabilities at shoot-look-shoot mode of shooting target 2

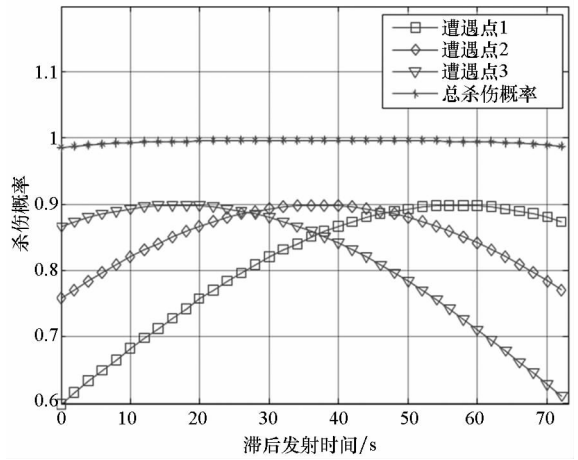


图 5 对目标 2 在射击-射击模式下滞后发射时间与杀伤概率关系图(发射间隔为 20s)

Fig. 5 Relationship between interceptors' delayed launch time and kill probabilities at shoot-shoot mode of shooting target 2 (shot interval is 20s)

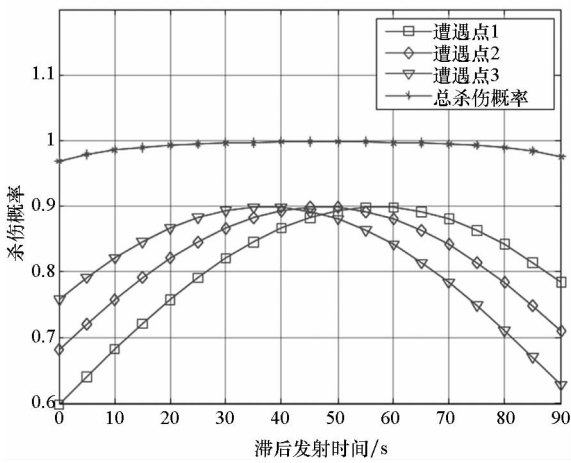


图 4 对目标 2 在射击-射击模式下滞后发射时间与杀伤概率关系图(发射间隔为 10s)

Fig. 4 Relationship between interceptors' delayed launch time and kill probabilities at shoot-shoot mode of shooting target 2 (shot interval is 10s)

表 4 对目标 2 在不同发射间隔下的遭遇点预测结果  
Tab. 4 Forecast results at different shot interval of shooting target 2

	发射间隔 10s	发射间隔 20s
点 1	(129.349 7,0,103.293 2)	(133.653 5,0,110.747 7)
点 2	(124.278 0,94.508 8)	(123.513 6,0, 93.189 6)
点 3	(119.194 7,0, 85.705 3)	(113.332 8,0, 75.551 3)
1 概率	0.899 1	0.897 4
2 概率	0.887 4	0.884 1
3 概率	0.891 3	0.889 7
总概率	0.990 7	0.990 3
最优 $\Delta t$	45s	36s

限于篇幅,对目标 3 的遭遇点参数计算不再写出。通过实例分析,验证了模型的可用性,此外,还可以看出最大概率拦截准则下对目标的杀伤概率的确有所提高,射击-射击模式比射击-观察-射击模式的杀伤概率高,其原因在于发射间隔选取比较灵活。

### 7 结论

遭遇点预测本质上是一个计算过程,此外,目标指示信息的精度和弹道预测的准确度对准确预测遭遇点至关重要。主要在多准则下建立了反导作战单个核多个遭遇点预测模型,部分方法具有一定探索性,下一步还要考虑拦截弹的导引规律去确定发射点到遭遇点之间拦截弹的轨迹,而不是假设成直飞逼近。此外,计算方程的变量较多,求解问题得进一步研究。

### 参考文献 (References)

[1] 荆武兴,李罗钢,高长生.反导拦截飞行方案及时间窗口快速搜索算法[J].系统工程与电子技术,2013,35(6):1256-1261.  
JING Wuxing, LI Luogang, GAO Changsheng. Fast search algorithm of flight program and launch time window for interception anti-missile [J]. Systems Engineering and Electronic, 2013, 35(6):1256-1261. (in Chinese)

[2] 王君,周林,雷虎民.地空导弹与空中目标遭遇点预测模型和算法[J].系统仿真学报,2009,21(1):80-83.  
WANG Jun, ZHOU Lin, LEI Humin. Forecast model and arithmetic on hit point of ground-to-air missile and aerial target[J]. Journal of System Simulation, 2009, 21(1):80-83. (in Chinese)



- [3] 张友安,马国欣,万宇. 一种弹目遭遇点预测方法[J]. 海军航空工程学院学报,2011,26(5):513-516.  
ZHANG Youan, MA Guoxin, WAN Yu. A method to predict impact point of missile and target [J]. Journal of Naval Aeronautical Engineering Institute, 2011, 26(5):513-516. (in Chinese)
- [4] 万雨君,刘鲁华,陈克俊,等. 基于预测命中点的机动目标最优制导方法[J]. 国防科技大学学报,2012,34(5):21-25.  
WAN Yujun, LIU Luhua, CHEN Kejun, et al. Optimal guidance law based on hit point forecast for maneuvering target[J]. Journal of National University of Defense Technology, 2012, 34(5):21-25. (in Chinese)
- [5] Dwivedi P N, Bhale P G, Bhattacharyya A, et al. Generalized state estimation and model predictive guidance for spiraling and ballistic targets [J]. Journal of Guidance, Control, and Dynamics, 2014, 37(1):243-264.
- [6] Liu Y F, Qi N M, Tang Z W. Effects of divert-thrusters on homing performance of endo-atmospheric interceptors [J]. Journal of Optimization Theory and Applications, 2013, 156(2):345-364.
- [7] Zheng L L. Ballistic missile interception from UCAV [D]. USA:Naval Postgraduate School,2011.
- [8] Kui Z L, Fang Y F, Chuan T X. Research on compound control technique with lateral impulsive thrust vector for air defence missile [C]. 32nd Chinese Control Conference (CCC 2013), 2013, 10: 4386-4390.
- [9] 全杰. 反导指挥决策的拦截可行性建模[J]. 指挥控制与仿真,2013,35(5):22-26.  
QUAN Jie. Intercepting feasibility moded of antimissile commanding decision [J]. Command Control &Simulation, 2013, 35(5):22-26. (in Chinese)
- [10] 季军亮,陈杰生,刘飞. 匈牙利法的末段两层火力反 TBM 目标分配优化[J]. 火力与指挥控制, 2011, 36(10): 192-195.  
JI Junliang, CHEN Jiesheng, LIU Fei. Optimization of targets assignment of dual-layer fire ATBM in reentry phase based on hungary [J]. Fire Control and Command Control, 2011, 36(10): 192-195. (in Chinese)
- [11] 符小卫,李金亮,高晓光. 防空威胁联网建模与分析[J]. 兵工学报,2013,34(7):904-909.  
FU Xiaowei, LI Jinliang, GAO Xiaoguang. Modeling and analysing of air-defense threat netting [J]. Acta Armamentarii, 2013, 34(7):904-909. (in Chinese)
- [12] 徐豫新,王树山,马晓飞. Monte-Carlo 法在武器系统射击参量设计中的应用[J]. 系统工程理论与实践,2012, 32(4): 854-859.  
XU Yuxin, WANG Shushan, MA Xiaofei. Application of Monte-Carlo method on the design of ammunition system firing parameters [J]. Systems Engineering-Theory & Practice, 2012, 32(4):854-859. (in Chinese)
- [13] 陈红彬,钱林方. 中口径火炮提前发射修正弹反导能力研究[J]. 弹道学报,2012,24(4):47-50.  
CHEN Hongbin, QIAN Linfang. Research on leading launching trajectory correction projectile of medium caliber gun against missile [J]. Journal of Ballistics, 2012, 24(4): 47-50. (in Chinese)
- [14] 段锁力. 末段高层反导火力单元任务规划关键问题研究[D]. 西安:空军工程大学,2012.  
DUAN Suoli. Research on the pivotal mission planning problems of the terminal high altitude anti-missile weapon [D]. Xi'an: Air Force Engineering University, 2012. (in Chinese)