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具有混合执行器故障和多未知控制方向的多智能体编队系统自适应协同容错控制*

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摘 要:针对非线性多智能体具有混合执行器故障和多未知控制方向的问题,基于鲁棒自适应模糊技术,提出新颖的协同容错控制方法。采用分段 Nussbaum 函数解决了多未知控制方向;基于鲁棒自适应模糊技术解决了系统中的非线性不确定问题;引入一阶滑模微分器与自适应反步技术结合,用于获得虚拟控制律的一阶导数,并结合鲁棒有界方法,用于提高所设计的自适应协同控制器的收敛速度和追踪精度;基于代数图论,建立了智能体系统的误差模型,设计了分布式一致协同容错控制器,构建了李雅普诺夫候选函数,证明了所设计的控制器能够使得系统稳定。通过对比仿真实例,验证了所提方法的有效性,为工程实践提供了理论支撑。

关键词:混合执行器故障;多未知控制方向;多智能体编队系统;鲁棒自适应模糊技术;一阶滑模微分器;协同容错

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Adaptive collaborative fault tolerance for multi-agent formation system with hybrid actuator faults and multiple unknown control directions

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Abstract: The cooperative fault-tolerant for the nonlinear multi-agent system with hybrid actuator faults and multiple unknown control directions were investigated, based on the RAFSMT(robust adaptive fuzzy sliding mode technology). The piecewise Nussbaum function was used to solve the multiple unknown control directions, and the robust adaptive fuzzy technology was used to solve the nonlinear uncertainty in the system. A first-order sliding mode differentiator was introduced, and it was combined with the adaptive backstepping technique to obtain the first-order derivative of the virtual control law. At the same time, a robust bounded method was adopted to improve the convergence speed and tracking accuracy of the designed adaptive cooperative controller. Based on the algebraic graph theory, the error model of the agent system was established, the distributed consensus cooperative fault-tolerant controller was designed, and the Lyapunov candidate function was constructed to prove that the proposed controller can make the system stable. The effectiveness of the proposed method is verified by the comparison of simulation examples, which provides theoretical support for engineering practice.

Keywords: hybrid actuator faults; multiple unknown control directions; multi-agent formation system; robust adaptive fuzzy technology; a first-order sliding mode differentiator; collaborative fault-tolerance

过去十几年,多智能体系统的协同容错问题吸引了大批学者的广泛研究[1-6]。需要指出的是,随着多智能体系统复杂程度的不断加深以及控制规模的不断扩大,其不可避免地具有更多的未知不确定性、多变量特性以及更频繁的系统故障,从而严重影响编队系统的高性能动态特性。例如,当多智能体系统的模型参数不准确时,会直接影响控制效果。同时,如果不能及时排除系统故障,尤其是执行器故障,就可能使整个系统失效、瘫痪以及造成人员、财产的巨大损失,甚至导致灾难性后果。尽管这些问题过去一段时间已有了很多研究成果,但这些难题依然是开放的。

为了补偿系统中执行器故障和解决未知控制方向的问题,已有多种基于现代控制理论的方法

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用于解决上述问题。目前,执行器故障问题大多基于某种故障模型进行研究[7-15]。系统未知控制方向问题则采用 Nussbaum 函数法,用于解决未知控制增益符号问题[16]。文献[17-24]采用传统的 Nussbaum 函数方法解决了一种非线性系统的未知控制增益问题,但其均假设了系统中的所有控制增益均未知。文献[25]采用分数阶 Nussbaum 函数解决混沌系统的不同多未知控制增益问题。文献[8,26]用一组 Nussbaum 函数方法解决了非线性系统的多个不同未知控制增益问题。文献[9]将 Nussbaum 函数与滤波器相结合,利用滤波器解决 Nussbaum 函数缺少足够反馈信号的问题。文献[27-28]用分段 Nussbaum 函数方法解决部分控制增益未知的情况,但其假设未知部分的控制增益为正数。然而,上述方法仍存在问题,有些方法解决的是非线性系统仅存在同方向的部分控制增益未知问题,或者同方向的控制增益完全未知的问题,这两种情况均不能反映实际系统所面临的问题。在实际系统中,部分控制增益未知,部分控制增益已知。综上可知,目前鲜有同时研究具有混合执行器故障和部分不同未知控制增益问题的研究。

基于以上研究不足与现有研究的挑战,本文针对一类有向通信拓扑下,具有混合执行器故障和部分不同未知控制方向的非线性多智能体系统的"领航者-跟随者"一致性问题,提出了一种新颖的鲁棒自适应模糊协同容错控制方案。

1 预备知识

1.1 图论

图论是建立多智能体之间网络拓扑结构的数学工具。图 $\bar{G}$ 代表任意两个智能体节点以及它们之间的边所构成的有向图。

假设1:图 $\bar{G}$ 存在一个有向生成树,并且根节点为领航者智能体。

假设2:有向图 $\bar{G}$ 的根节点能够直接获得参考轨迹的所有信息,即 $b_i=1$,反之 $b_i=0$。

假设3:领航者的状态及其导数均为有界的。

1.2 Nussbaum 函数

如果 $N(\tau):R\to R$ 具有如式(1)所示的性质,则称 $N(\tau)$ 为 Nussbaum 函数。

$$\begin{cases}\lim\limits_{\omega\to\infty}\sup\dfrac{1}{\omega}\displaystyle\int_0^{\omega}N(\tau)\,\mathrm{d}\tau=+\infty\\ \lim\limits_{\omega\to-\infty}\inf\dfrac{1}{\omega}\displaystyle\int_0^{\omega}N(\tau)\,\mathrm{d}\tau=-\infty\end{cases}\tag{1}$$

同时,为了解决多未知控制方向问题,给出如式(2)所示的定义。

$$N_R(\zeta)=\begin{cases}N_R^1(\zeta),\ 方向已知\\ N_R^2(\zeta),\ 方向未知\end{cases}\tag{2}$$

式中,$N_R^1(\zeta)=-\varphi\exp(\zeta^2/2)(\zeta^2+2)\cos[(\pi/2)\zeta]$,$N_R^2(\zeta)=-\exp(\zeta^2/2)\zeta$,此处 φ 是一个正实数。

引理1[22] 如果 $V(t)$ 是定义在 $[0,\zeta)$ 的正定函数,且 $V(0)$ 是有界的,有如式(3)所示不等式成立。

$$V_i(t)\leqslant\sum_{l=1}^{n_i}\int_0^t g_{i,l}(\zeta)\psi_{i,l}N_R(\xi_{i,l}(\zeta))\dot{\xi}_{i,l}(\zeta)\,\mathrm{d}\zeta+\sum_{l=1}^{n_i}\int_0^t\nu_i\dot{\xi}_{i,l}(\zeta)\,\mathrm{d}\zeta+\vartheta\tag{3}$$

式中:$i=1,2,\cdots,N$;ϑ 为非负常数;$g_{i,l}(\zeta)$ 和 $\psi_{i,l}$ 有界,而且 $\sum_{l=1}^{n_i}\int_0^t[g_{i,l}(\zeta)\psi_{i,l}N_R(\xi_{i,l}(\zeta))+\nu_i]\dot{\xi}_{i,l}(\zeta)\,\mathrm{d}\zeta$ 也是有界的。

1.3 模糊逻辑系统

引理2 对于 $\forall\varepsilon>0$,存在紧集 Ω_Z、连续函数 $f(\boldsymbol{Z})$ 和模糊逻辑系统,则有:

$$\sup_{Z\in\Omega_Z}|f(\boldsymbol{Z})-\boldsymbol{W}^{*\mathrm{T}}\boldsymbol{\Theta}(\boldsymbol{Z})|\leqslant\bar{\varepsilon}\tag{4}$$

式中,$\boldsymbol{W}^*=\arg\min\limits_{W\in\mathbf{R}^l}[\sup\limits_{x\in\Omega_Z}|f(\boldsymbol{Z})-f(\boldsymbol{Z},\boldsymbol{W}^*)|]$ 是最优参数向量,$\mathbf{R}^l$ 表示实数,$\boldsymbol{\Theta}(\boldsymbol{Z})$ 表示模糊基函数向量。

1.4 其他相关理论和引理

引理3[29] 对于任意 $x\in R^n$,且 $\bar{\lambda}>0$,则有:

$$0\leqslant|x|-x\tanh\left(\frac{x}{\bar{\lambda}}\right)\leqslant0.2785\bar{\lambda}\tag{5}$$

引理4[30] 对于 $\forall x\geqslant0$,$\forall y\geqslant0$,$p\in\mathbf{R}^+$,$q\in\mathbf{R}^+$,同时满足 $1/p+1/q=1$:

$$xy\leqslant\frac{x^p}{p}+\frac{y^q}{q}\tag{6}$$

引理5[25] 引入一阶滑模微分器的设计,如式(7)所示:

$$\begin{cases}\dot{v}_0=\bar{\omega}_0=-a_0|v_0-h(t)|^{\frac{1}{2}}\operatorname{sign}(v_0-h(t))+v_1\\ \dot{v}_1=-a_1\operatorname{sign}(v_1-\bar{\omega}_0)\end{cases}\tag{7}$$

其中:$v_0,v_1,\bar{\omega}_0$ 均表示系统式(7)的状态变量;a_0 和 a_1 是一阶滑模微分器的设计参数;$h(t)$ 是未知函数。$\bar{\omega}_0$ 能够估计 $\dot{h}(t)$ 的精确值。此外,$v_0-h(t_0)$ 和 $\bar{\omega}_0-\dot{h}(t_0)$ 的初始导数有界。

2 问题描述

2.1 系统模型

考虑同构智能体编队系统每个子系统的模型

可描述为:

$$\begin{cases}\dot{x}_{i,k}=f_{i,k}(\bar{\boldsymbol{x}}_{i,k})+g_{i,k}(\bar{\boldsymbol{x}}_{i,k})x_{i,k+1}\\ \dot{x}_{i,n_i}=f_{i,n}(\bar{\boldsymbol{x}}_{i,n})+g_{i,k}(\bar{\boldsymbol{x}}_{i,n})u_i^F\\ y_i=x_{i,1}\end{cases} \tag{8}$$

其中,$1\leqslant i\leqslant n$,$\bar{\boldsymbol{x}}_{i,k}=[x_{i,1},\cdots,x_{i,k}]^{\mathrm{T}}\in\mathbf{R}^k$,$1\leqslant k\leqslant n_i-1$,$\boldsymbol{x}_i=[x_{i,1},\cdots,x_{i,n_i}]^{\mathrm{T}}\in\mathbf{R}^{n_i}$表示第 i 个跟随智能体的状态变量,$y_i\in\mathbf{R}$ 为第 i 个跟随智能体的输出,$f_{i,k}(\cdot)$和 $g_{i,k}(\cdot)$为未知连续非线性光滑函数,u_i^F 表示控制信号。

2.2 故障模型

为了更好研究实际系统中含有的混合执行器故障,将系统模型式(8)中的故障模型类型进行分类,如式(9)所示。

$$u_i^F(t)=[1-k(\rho)]u_i(t)+\kappa\tilde{u}(t) \tag{9}$$

式中,$u_i^F(t)$和 $u_i(t)$表示第 i 个智能体输入信号和执行器输入信号,$\tilde{u}_i(t)$表示第 i 个执行器发生故障时的输入信号,为了不失一般性,将故障分为 4 类,见表 1。

表 1　执行器故障类型

Tab. 1　Actuator fault types

类型	数值
无故障	$\rho_1=\rho_2=0,\kappa_1=0$
部分失效故障	$0<\rho_1<\rho_2<1,\kappa_1=0$
偏置故障	$0<\rho_1<\rho_2<1,\kappa_1=1$
卡死故障	$\rho_i=1,\kappa_i=1$

在表 1 中,执行器处于偏置故障时,$\tilde{u}(t)$是有界的,即$\|\tilde{u}(t)\|\leqslant\delta_d$,其中 δ_d 是已知的正数,而且偏置故障是未知时变有界的。

3 控制器设计

在本节中,针对具有混合执行器故障式(8)和多未知控制方向式(2)的非线性多智能体系统,基于鲁棒自适应模糊技术设计一种新颖的协同容错控制器。

定义同步误差:

$$\begin{cases}e_{i,1}=\sum_{j\in N_i}a_{ij}(y_i-y_j)+b_i(y_i-y_r)\\ \alpha_{i,k-1}^*=\alpha_{i,k-1}-\beta_{i,k-1}\tanh(\lambda_{i,k-1}e_{i,k-1})\end{cases} \tag{10}$$

其中:a_{ij} 表示第 i 个智能体与第 j 个智能体之间通信的权重系数;b_i 表示领航者和跟随者的节点增益;$\boldsymbol{e}_1=[e_{1,1},e_{2,1},\cdots,e_{N-1}]^{\mathrm{T}}\in\mathbf{R}^N$。

根据图论,式(10)可进一步重写为:

$$\boldsymbol{e}_1=(\bar{\boldsymbol{L}}+\boldsymbol{B})\delta \tag{11}$$

式中,$\boldsymbol{e}_1=[e_{1,1},e_{2,1},\cdots,e_{N-1}]^{\mathrm{T}}\in\mathbf{R}^N$,$\delta=y_i-y_r$ 表示第 i 个跟随者的跟随误差。定义$\bar{\boldsymbol{\delta}}=\bar{\boldsymbol{y}}-\bar{\boldsymbol{y}}_r$,此处 $\bar{\boldsymbol{y}}=[y_1,y_2,\cdots,y_N]^{\mathrm{T}}$,$\bar{\boldsymbol{y}}_r=[y_r,y_r,\cdots,y_r]^{\mathrm{T}}$,$\|\delta\|\leqslant\dfrac{\|\boldsymbol{e}_1\|}{\bar{\boldsymbol{\delta}}(\bar{\boldsymbol{L}}+\boldsymbol{B})}$,其中$\bar{\boldsymbol{\delta}}(\bar{\boldsymbol{L}}+\boldsymbol{B})$是$\bar{\boldsymbol{L}}+\boldsymbol{B}$的最小奇异值,$\bar{\boldsymbol{L}}+\boldsymbol{B}$为非奇异矩阵,$\boldsymbol{B}=[b_1,b_2,\cdots,b_N]^{\mathrm{T}}=\mathrm{diag}(b_1,b_2,\cdots,b_N)\in\mathbf{R}^{N\times N}$。

联立式(8)和式(10),采用反步技术,可得:

$$e_{i,1}=(A_i+b_i)g_{i,1}(\bar{x}_{i,1})x_{i,2}+F_{i,1}(\boldsymbol{Z}_{i,1}) \tag{12}$$

式中,$\boldsymbol{Z}_{i,1}=[x_{i,1},x_{j,1},x_{j,2}]^{\mathrm{T}}(j\in\bar{N}_i)$,$\sum_{j\in N_i}a_{ij}=A_i F_{i,1}(\boldsymbol{Z}_{i,1})=-\sum_{j\in N_i}a_{ij}[f_{j,1}(\bar{x}_{j,1})+g(\bar{x}_{j,1})x_{j,2}]+(\sum_{j\in N_i}a_{ij}+b_i)f_{i,1}(\bar{x}_{i,1})-b_i\dot{y}_r$。

利用模糊逻辑系统对 $F_{i,1}(\boldsymbol{Z}_{i,1})$逼近,可得:

$$F_{i,1}(\boldsymbol{Z}_{i,1})=\boldsymbol{W}_{i,1}^{*\mathrm{T}}\boldsymbol{\Theta}_{i,1}(\boldsymbol{Z}_{i,1})+\varepsilon_{i,1}(\boldsymbol{Z}_{i,1}) \tag{13}$$

式中,$\varepsilon_{i,1}(\boldsymbol{Z}_{i,1})\leqslant\bar{\varepsilon}_{i,1}(\boldsymbol{Z}_{i,1})$,$\boldsymbol{\Theta}_{i,1}(\boldsymbol{Z}_{i,1})$表示模糊径向基函数向量,$\boldsymbol{W}_{i,1}^*$ 表示理想参数向量,$\varepsilon_{i,1}(\boldsymbol{Z}_{i,1})$表示逼近误差。

根据 Young′s 不等式,联立式(12)和引理 3,可进一步得出:

$$\begin{aligned}e_{i,1}F_{i,1}&=e_{i,1}[\boldsymbol{W}_{i,1}^{*\mathrm{T}}\boldsymbol{\Theta}_{i,1}(\boldsymbol{Z}_{i,1})+\varepsilon_{i,1}(\boldsymbol{Z}_{i,1})]\\&\leqslant|e_{i,1}|\|\boldsymbol{W}_{i,1}^{*\mathrm{T}}\|\|\boldsymbol{\Theta}_{i,1}\|+|e_{i,1}\varepsilon_{i,1}|\\&\leqslant e_{i,1}\eta_{i,1}\|\boldsymbol{\Theta}_{i,1}\|\tanh\left(\frac{e_{i,1}\|\boldsymbol{\Theta}_{i,1}\|}{\gamma_{i,1}}\right)+\\&\quad\varepsilon\eta_{i,1}\gamma_{i,1}+\frac{1}{2}e_{i,1}^2+\frac{1}{2}\bar{\varepsilon}_{i,1}^2\end{aligned} \tag{14}$$

构建 Lyapunov 函数:

$$V_{i,1}=\frac{1}{2}e_{i,1}^2+\frac{1}{2\boldsymbol{\Gamma}_{i,1}}\tilde{\eta}_{i,1}^2 \tag{15}$$

式中,$\boldsymbol{\Gamma}_{i,1}$ 表示正定矩阵,$\eta_{i,1}$ 表示待估计参数。

对式(15)求导,结合式(12),可得:

$$\begin{aligned}\dot{V}_{i,1}&=e_{i,1}\dot{e}_{i,1}-\eta_{i,1}\boldsymbol{\Gamma}_{i,1}^{-1}\dot{\hat{\eta}}_{i,1}\\&=e_{i,1}(A_{ij}+b_i)g_{i,1}(\bar{x}_{i,1})(e_{i,2}+\alpha_{i,1}^*)+\\&\quad e_{i,1}F_{i,1}(\boldsymbol{Z}_{i,1})-\eta_{i,1}\boldsymbol{\Gamma}_{i,1}^{-1}\dot{\hat{\eta}}_{i,1}\\&\leqslant(A_{ij}+b_i)g_{i,1}(\bar{x}_{i,1})e_{i,1}e_{i,2}+\\&\quad(A_{ij}+b_i)g_{i,1}(\bar{x}_{i,1})e_{i,1}\alpha_{i,1}^*+\\&\quad e_{i,1}\eta_{i,1}\|\boldsymbol{\Theta}_{i,1}\|\tanh\left(\frac{e_{i,1}\|\boldsymbol{\Theta}_{i,1}\|}{\gamma_{i,1}}\right)+\varepsilon\eta_{i,1}\gamma_{i,1}+\\&\quad\frac{1}{2}e_{i,1}^2+\frac{1}{2}\bar{\varepsilon}_{i,1}^2-\eta_{i,1}\boldsymbol{\Gamma}_{i,1}^{-1}\dot{\hat{\eta}}_{i,1}\\&\leqslant(A_{ij}+b_i)g_{i,1}(\bar{x}_{i,1})e_{i,1}e_{i,2}+\\&\quad(A_{ij}+b_i)g_{i,1}(\bar{x}_{i,1})e_{i,1}[\alpha_{i,1}-\beta_{i,1}\tanh(\lambda_{i,1}e_{i,1})]+\\&\quad e_{i,1}\eta_{i,1}\|\boldsymbol{\Theta}_{i,1}\|\tanh\left(\frac{e_{i,1}\|\boldsymbol{\Theta}_{i,1}\|}{\gamma_{i,1}}\right)+\varepsilon\eta_{i,1}\gamma_{i,1}+\end{aligned}$$

$$\frac{1}{2}e_{i,1}^2+\frac{1}{2}\bar{\varepsilon}_{i,1}^2-\eta_{i,1}\boldsymbol{\Gamma}_{i,1}^{-1}\dot{\hat{\eta}}_{i,1}$$
$$\leqslant (A_{ij}+b_i)g_{i,1}(\bar{x}_{i,1})e_{i,1}e_{i,2}+(A_{ij}+b_i)g_{i,1}(\bar{x}_{i,1})e_{i,1}\alpha_{i,1}-(A_{ij}+b_i)g_{i,1}(\bar{x}_{i,1})e_{i,1}\beta_{i,1}\tanh(\lambda_{i,1}e_{i,1})+e_{i,1}\eta_{i,1}\|\boldsymbol{\Theta}_{i,1}\|\tanh\left(\frac{e_{i,1}\|\boldsymbol{\Theta}_{i,1}\|}{\gamma_{i,1}}\right)+\varepsilon\eta_{i,1}\gamma_{i,1}+\frac{1}{2}e_{i,1}^2+\frac{1}{2}\bar{\varepsilon}_{i,1}^2-\eta_{i,1}\boldsymbol{\Gamma}_{i,1}^{-1}\dot{\hat{\eta}}_{i,1} \tag{16}$$

由式(16)可得虚拟控制律及对应的更新律。

$$\alpha_{i,1}=(A_{ij}+b_i)^{-1}N(\zeta_{i,1})\left[\hat{\eta}_{i,1}\|\boldsymbol{\Theta}_{i,1}\|\tanh\left(\frac{e_{i,1}\|\boldsymbol{\Theta}_{i,1}\|}{\gamma_{i,1}}\right)+e_{i,1}-\beta_{i,1}\tanh(\lambda_{i,1}e_{i,1})\right] \tag{17}$$

$$\dot{\zeta}_{i,1}=(A_{ij}+b_i)\left[e_{i,1}\hat{\eta}_{i,1}\|\boldsymbol{\Theta}_{i,1}\|\tanh\left(\frac{e_{i,1}\|\boldsymbol{\Theta}_{i,1}\|}{\gamma_{i,1}}\right)+e_{i,1}^2-e_{i,1}\beta_{i,1}\tanh(\lambda_{i,1}e_{i,1})\right] \tag{18}$$

$$\dot{\hat{\eta}}_{i,1}=\boldsymbol{\Gamma}_{i,1}\left[e_{i,1}\hat{\eta}_{i,1}\|\boldsymbol{\Theta}_{i,1}\|\tanh\left(\frac{e_{i,1}\|\boldsymbol{\Theta}_{i,1}\|}{\gamma_{i,1}}\right)-\sigma_{i,1}\|e\|\hat{\eta}_{i,1}\right] \tag{19}$$

将式(17)~(19)代入式(16),可得:

$$\dot{V}_{i,1}\leqslant(A_{ij}+b_i)\left(\frac{1}{4}e_{i,1}^2+g_{i,1}^2e_{i,2}^2\right)+g_{i,1}N(\zeta_{i,1})\dot{\zeta}_{i,1}+\dot{\zeta}_{i,1}-(A_{ij}+b_i)e_{i,1}^2+\varepsilon\eta_{i,1}\gamma_{i,1}+\frac{1}{2}\bar{\varepsilon}_{i,1}^2+\boldsymbol{\Gamma}_{i,1}^{-1}\tilde{\eta}_{i,1}\left[e_{i,1}\boldsymbol{\Gamma}_{i,1}\|\boldsymbol{\Theta}_{i,1}\|\tanh\left(\frac{e_{i,1}\|\boldsymbol{\Theta}_{i,1}\|}{\gamma_{i,1}}\right)-\dot{\hat{\eta}}_{i,1}\right] \tag{20}$$

将式(18)代入式(20),可得:

$$\dot{V}_{i,1}\leqslant-\frac{3}{4}(A_{ij}+b_i)e_{i,1}^2+(A_{ij}+b_i)g_{i,1}^2e_{i,2}^2+g_{i,1}N(\zeta_{i,1})\dot{\zeta}_{i,1}+\dot{\zeta}_{i,1}+\boldsymbol{\Gamma}_{i,1}^{-1}\tilde{\eta}_{i,1}\sigma_{i,1}\|e\|\hat{\eta}_{i,1}+\varepsilon\eta_{i,1}\gamma_{i,1}+\frac{1}{2}\bar{\varepsilon}_{i,1}^2$$
$$\leqslant-\frac{3}{4}(A_{ij}+b_i)e_{i,1}^2+(A_{ij}+b_i)g_{i,1}^2e_{i,2}^2+g_{i,1}N(\zeta_{i,1})\dot{\zeta}_{i,1}+\dot{\zeta}_{i,1}-\frac{\sigma_{i,1}\|e\|}{2\boldsymbol{\Gamma}_{i,1}}\tilde{\eta}_{i,1}^2+\frac{\sigma_{i,1}\|e\|}{2\boldsymbol{\Gamma}_{i,1}}\eta_{i,1}^2+\varepsilon\eta_{i,1}\gamma_{i,1}+\frac{1}{2}\bar{\varepsilon}_{i,1}^2 \tag{21}$$

令 $\Xi_1=\frac{\sigma_{i,1}\|e\|}{2\boldsymbol{\Gamma}_{i,1}}\eta_{i,1}^2+\varepsilon\eta_{i,1}\gamma_{i,1}+\frac{1}{2}\bar{\varepsilon}_{i,1}^2$ 和 $c_{i,1}=\min\left\{\frac{3}{2},\varepsilon,\sigma_{i,1}\right\}$,根据引理1,则有:

$$\frac{\mathrm{d}}{\mathrm{d}t}(\dot{V}_{i,1}\mathrm{e}^{c_{i,1}t})\leqslant-c_{i,1}V_{i,1}+\Xi_1+g_{i,1}N(\zeta_{i,1})\dot{\zeta}_{i,1}+\dot{\zeta}_{i,1}+(A_{ij}+b_i)g_{i,1}^2e_{i,2}^2 \tag{22}$$

式(22)两边同乘以 $\mathrm{e}^{c_{i,1}t}$,可得:

$$\frac{\mathrm{d}}{\mathrm{d}t}(\dot{V}_{i,1}\mathrm{e}^{c_{i,1}t})\leqslant\Xi_1\mathrm{e}^{c_{i,1}t}+g_{i,1}N(\zeta_{i,1})\dot{\zeta}_{i,1}\mathrm{e}^{c_{i,1}t}+\dot{\zeta}_{i,1}\mathrm{e}^{c_{i,1}t}+(A_{ij}+b_i)g_{i,1}^2e_{i,2}^2\mathrm{e}^{c_{i,1}t} \tag{23}$$

对式(23)两边同时积分,则有:

$$V_{i,1}(t)\leqslant\frac{\Xi_1}{c_{i,1}}+\mathrm{e}^{-c_{i,1}t}\int_0^t[g_{i,1}N(\zeta_{i,1})+1]\dot{\zeta}_{i,1}\mathrm{e}^{c_{i,1}t}\mathrm{d}t+\mathrm{e}^{-c_{i,1}t}(A_{ij}+b_i)\int_0^t[g_{i,1}N(\zeta_{i,1})+1]\dot{\zeta}_{i,1}\mathrm{e}^{c_{i,1}t}\mathrm{d}t \tag{24}$$

第 k 步($2\leqslant k\leqslant n_i-1$):定义误差,如式(25)所示。

$$e_{i,k}=x_{i,k}-\alpha_{i,k-1}^* \tag{25}$$

对式(25)求导,可得:

$$\dot{e}_{i,k}=\dot{x}_{i,k}-\dot{\alpha}_{i,k-1}^*=g_{i,k}(\bar{x}_{i,k})x_{i,k+1}+f_{i,k}(\bar{x}_{i,k})-\dot{\alpha}_{i,k-1}^* \tag{26}$$

引入一阶滑模微分器,估计虚拟控制律 $\dot{\alpha}_{i,k-1}^*$,考虑如式(27)所示的系统。

$$\begin{cases}\dot{v}_{i0,k}=\bar{\omega}_{i0,k}=v_{i0,k}-a_{i0,k}\left|v_{i0,k}-\alpha_{i,k-1}^*\right|^{1/2}\mathrm{sign}(v_{i0,k}-\alpha_{i,k-1}^*)\\ \dot{v}_{i1,k}=-a_{i1,k}\mathrm{sign}(v_{i1,k}-\bar{\omega}_{i0,k})\end{cases} \tag{27}$$

其中:$v_{i0,k}$,$v_{i1,k}$和 $\bar{\omega}_{i0,k}$ 均为系统的状态变量;$a_{i0,k}$,$a_{i1,k}$均为待估计的正参数。

于是,可得 $\dot{\alpha}_{i,k-1}^*$ 表达式。

$$\dot{\alpha}_{i,k-1}^*=\bar{\omega}_{i0,k-1}+d_{i,k-1} \tag{28}$$

式中,$d_{i,k-1}$是一阶滑模微分器的估计误差。通过引理3,可知:$|d_{i,k-1}|\leqslant\bar{d}_{i,k-1}$且 $d_{i,k-1}>0$。

将式(28)代入式(26),可得:

$$\dot{e}_{i,k}=g_{i,k}(\bar{x}_{i,k})(e_{i,k+1}+\alpha_{i,k}^*)+f_{i,k}(\bar{x}_{i,k})-\bar{\omega}_{i0,k-1}-d_{i,k-1} \tag{29}$$

令 $F_{i,k}(\boldsymbol{Z}_{i,k})=f_{i,k}(\bar{x}_{i,k})-\bar{\omega}_{i0,k-1}$,则式(29)可进一步重写为:

$$\dot{e}_{i,k}=g_{i,k}(\bar{x}_{i,k})(e_{i,k+1}+\alpha_{i,k})+F_{i,k}(\boldsymbol{Z}_{i,k})-d_{i,k-1} \tag{30}$$

构建 Lyapunov 函数:

$$V_{i,k}=V_{i,k-1}+\frac{1}{2}e_{i,k}^2+\frac{1}{2\boldsymbol{\Gamma}_{i,k}}\tilde{\eta}_{i,2}^2 \tag{31}$$

对式(31)求导,结合式(29)和引理4可得:

$$\dot{V}_{i,k}=V_{i,k-1}+e_{i,k}\dot{e}_{i,k}-\tilde{\eta}_{i,k}\boldsymbol{\Gamma}_{i,k}^{-1}\dot{\hat{\eta}}_{i,k}$$
$$\leqslant\dot{V}_{i,k-1}+e_{i,k}[g_{i,k}(\bar{x}_{i,k})x_{i,k+1}+F_{i,k}(\boldsymbol{Z}_{i,k})]-\tilde{\eta}_{i,k}\boldsymbol{\Gamma}_{i,k}^{-1}\dot{\hat{\eta}}_{i,k}$$
$$\leqslant\dot{V}_{i,k-1}+e_{i,k}[g_{i,k}(\bar{x}_{i,k})(e_{i,k+1}+\alpha_{i,k}^*)+$$

$$F_{i,k}(\boldsymbol{Z}_{i,k})] - \tilde{\eta}_{i,k}\boldsymbol{\Gamma}_{i,k}^{-1}\dot{\hat{\eta}}_{i,k}$$
$$\leqslant \dot{V}_{i,k-1} + e_{i,k}g_{i,k}(\bar{x}_{i,k})e_{i,k+1} + e_{i,k}g_{i,k}(\bar{x}_{i,k})[\alpha_{i,k} - \beta_{i,k}\tanh(\lambda_{i,k}e_{i,k})] + e_{i,k}\eta_{i,k}\|\boldsymbol{\Theta}_{i,k}\|\tanh\left(\frac{e_{i,k}\|\boldsymbol{\Theta}_{i,k}\|}{\gamma_{i,k}}\right) + \varepsilon_{i,k}\eta_{i,k}\gamma_{i,k} + \frac{1}{2}\bar{\varepsilon}_{i,k}^2 + \frac{1}{2}e_{i,k}^2 - e_{i,k}d_{i,k-1} - \tilde{\eta}_{i,k}\boldsymbol{\Gamma}_{i,k}^{-1}\dot{\hat{\eta}}_{i,k} \tag{32}$$

由式(32)可得虚拟控制律和更新律,如式(33)~(35)所示。

$$\alpha_{i,k} = N(\zeta_{i,k})\left[\hat{\eta}_{i,k}\|\boldsymbol{\Theta}_{i,k}\|\tanh\left(\frac{e_{i,k}\|\boldsymbol{\Theta}_{i,k}\|}{\gamma_{i,k}}\right) + e_{i,k} - \beta_{i,k}\tanh(\lambda_{i,k}e_{i,k})\right] \tag{33}$$

$$\dot{\zeta}_{i,k} = e_{i,k}^2 + e_{i,k}\hat{\eta}_{i,k}\|\boldsymbol{\Theta}_{i,k}\|\tanh\left(\frac{e_{i,k}\|\boldsymbol{\Theta}_{i,k}\|}{\gamma_{i,k}}\right) - e_{i,k}\beta_{i,k}\tanh(\lambda_{i,k}e_{i,k}) \tag{34}$$

$$\dot{\hat{\eta}}_{i,k} = \boldsymbol{\Gamma}_{i,k}\left[e_{i,k}\|\boldsymbol{\Theta}_{i,k}\|\tanh\left(\frac{e_{i,k}\|\boldsymbol{\Theta}_{i,k}\|}{\gamma_{i,k}}\right) - \sigma_{i,k}\|e\|\hat{\eta}_{i,k}\right] \tag{35}$$

将式(33)~(35)代入式(32),可得:

$$\dot{V}_{i,k} \leqslant -\frac{3}{4}(A_{ij}+b_i)e_{i,1}^2 - \frac{3}{4}\sum_{l=2}^{k-1}e_{i,l}^2 + (A_{ij}+b_i)g_{i,1}^2e_{i,2}^2 + \sum_{l=2}^{k-1}g_{i,l}^2e_{i,l+1}^2 + \sum_{l=1}^{k-1}\dot{\zeta}_{i,l}[g_{i,l}N(\zeta_{i,l})\dot{\zeta}_{i,l} + 1] + \sum_{l=1}^{k-1}\varepsilon_{i,l}\eta_{i,1}\gamma_{i,1} + \frac{1}{2}\sum_{l=1}^{k-1}\bar{\varepsilon}_{i,l}^2 + \sum_{l=1}^{k-1}\left[\frac{\sigma_{i,2}\|e\|}{2\boldsymbol{\Gamma}_{i,1}}(\eta_{i,2}^2 - \tilde{\eta}_{i,2}^2)\right] - \frac{3}{4}e_{i,k}^2 + g_{i,k}^2e_{i,k+1}^2 + \frac{1}{2}\bar{\varepsilon}_{i,k}^2 - e_{i,k}d_{i,k-1} + g_{i,k}N(\zeta_{i,k})\dot{\zeta}_{i,k} + \dot{\zeta}_{i,k} + \varepsilon_{i,k}\eta_{i,k}\gamma_{i,k} + \frac{\sigma_{i,k}\|e\|}{2\boldsymbol{\Gamma}_{i,k}}(\eta_{i,k}^2 - \tilde{\eta}_{i,k}^2)$$
$$\leqslant -\frac{3}{4}(A_{ij}+b_i)e_{i,1}^2 + (A_{ij}+b_i)g_{i,1}^2e_{i,2}^2 - \frac{3}{4}\sum_{l=2}^{k}e_{i,l}^2 + \sum_{l=2}^{k}g_{i,l}^2e_{i,l+1}^2 + \sum_{l=1}^{k}\dot{\zeta}_{i,l}[g_{i,l}N(\zeta_{i,l})\dot{\zeta}_{i,l} + 1] + \sum_{l=1}^{k}\varepsilon_{i,l}\eta_{i,1}\gamma_{i,1} + \frac{1}{2}\sum_{l=1}^{k}\bar{\varepsilon}_{i,l}^2 + \sum_{l=1}^{k}\left[\frac{\sigma_{i,2}\|e\|}{2\boldsymbol{\Gamma}_{i,1}}(\eta_{i,2}^2 - \tilde{\eta}_{i,2}^2)\right] - e_{i,k}d_{i,k-1} \tag{36}$$

令 $\varXi_k = \frac{\sigma_{i,k}\|e\|}{2\boldsymbol{\Gamma}_{i,k}}\eta_{i,k}^2 + \varepsilon\eta_{i,k}\gamma_{i,k} + \frac{1}{2}\bar{\varepsilon}_{i,k}^2 + \frac{1}{2}d_{i,k-1}^2$ 和 $c_{i,k} = \min\left\{\frac{3}{2},\varepsilon,\sigma_{i,k}\right\}$。根据引理 1,则有

$$\frac{\mathrm{d}}{\mathrm{d}t}(\dot{V}_{i,k}\mathrm{e}^{c_{i,k}t}) \leqslant -c_{i,k}V_{i,k} + \varXi_k + g_{i,k}N(\zeta_{i,k})\dot{\zeta}_{i,k} + \dot{\zeta}_{i,k} + (A_{ij}+b_i)g_{i,k}^2e_{i,k+1}^2 \tag{37}$$

式(37)两边同乘以 $\mathrm{e}^{c_{i,k}t}$,可得:

$$\frac{\mathrm{d}}{\mathrm{d}t}(\dot{V}_{i,k}\mathrm{e}^{c_{i,k}t}) \leqslant \varXi_k\mathrm{e}^{c_{i,k}t} + g_{i,k}N(\zeta_{i,k})\dot{\zeta}_{i,k}\mathrm{e}^{c_{i,k}t} + \dot{\zeta}_{i,k}\mathrm{e}^{c_{i,k}t} + (A_{ij}+b_i)g_{i,k}^2e_{i,k+1}^2\mathrm{e}^{c_{i,k}t} \tag{38}$$

对式(38)两边同时积分,可得:

$$V_{i,k}(t) \leqslant \frac{\varXi_k}{c_{i,k}} + \mathrm{e}^{-c_{i,k}t}\int_0^t[g_{i,k}N(\zeta_{i,k})+1]\dot{\zeta}_{i,k}\mathrm{e}^{c_{i,k}t}\mathrm{d}t + \mathrm{e}^{-c_{i,k}t}(A_{ij}+b_i)\int_0^t[g_{i,k}N(\zeta_{i,k})+1]\dot{\zeta}_{i,k}\mathrm{e}^{c_{i,k}t}\mathrm{d}t \tag{39}$$

第 N 步:定义误差,如式(40)所示。

$$e_{i,n_i} = x_{i,n_i} - \alpha_{i,n_i-1} \tag{40}$$

对式(40)求导,结合式(8),可得:

$$\dot{e}_{i,n_i} = \dot{x}_{i,n_i} - \dot{\alpha}_{i,n_i-1} = g_{i,n_i}(\bar{x}_{i,n_i})u_i + F_{i,n_i}(\boldsymbol{Z}_{i,n_i}) - \dot{\alpha}_{i,n_i-1} \tag{41}$$

同样,为了规避冗余的虚拟控制律求导计算,引入一阶滑模微分器技术,估计虚拟控制律 $\dot{\alpha}_{i,n_i-1}^*$,考虑如式(42)所示系统。

$$\begin{cases}\dot{v}_{i0,n_i} = \bar{\omega}_{i0,n_i-1} = v_{i0,n_i} \\ \quad - a_{i0,n_i}|v_{i0,n_i} - \alpha_{i,n_i-1}^*|^{1/2}\mathrm{sign}(v_{i0,n_i} - \alpha_{i,n_i-1}^*) \\ \dot{v}_{i1,n_i} = -a_{i1,n_i}\mathrm{sign}(v_{i1,n_i} - \bar{\omega}_{i0,n_i-1})\end{cases} \tag{42}$$

其中:$v_{i0,n_i}, v_{i1,n_i}, \bar{\omega}_{i0,n_i}$ 均为系统的状态变量;a_{i0,n_i}, a_{i1,n_i} 均为待估计的正参数。

于是可得 $\dot{\alpha}_{i,k-1}^*$ 表达式:

$$\dot{\alpha}_{i,n_i-1}^* = \bar{\omega}_{i0,n_i-1} + d_{i,n_i-1} \tag{43}$$

式中,d_{i,n_i-1} 是一阶滑模微分器的估计误差。通过引理 3,可知:$|d_{i,n_i-1}| \leqslant \bar{d}_{i,n_i-1}$ 且 $d_{i,n_i-1} > 0$。

进而,式(41)可进一步重写为:

$$\dot{e}_{i,n_i} = g_{i,n_i}(\bar{x}_{i,n_i})(e_{i,n_i+1} + \alpha_{i,n_i}^*) + f_{i,n_i}(\bar{x}_{i,n_i}) - \bar{\omega}_{i0,n_i-1} - d_{i,n_i-1} \tag{44}$$

令 $F_{i,n_i}(\boldsymbol{Z}_{i,n_i}) = f_{i,n_i}(\bar{x}_{i,n_i}) - \bar{\omega}_{i0,n_i-1}$,则有:

$$\dot{e}_{i,n_i} = g_{i,n_i}(\bar{x}_{i,n_i})(e_{i,n_i+1} + \alpha_{i,n}) + F_{i,n_i}(\boldsymbol{Z}_{i,n_i}) - d_{i,n_i-1} \tag{45}$$

根据 Young's 不等式,结合引理 4,则有:

$$e_{i,n_i}F_{i,n_i}(\boldsymbol{Z}_{i,n_i}) \leqslant e_{i,n_i}\eta_{i,n_i}\|\boldsymbol{\Theta}_{i,n_i}\|\tanh\left(\frac{e_{i,n_i}\|\boldsymbol{\Theta}_{i,n_i}\|}{\gamma_{i,n_i}}\right) + \varepsilon_{i,n_i}\eta_{i,n_i}\gamma_{i,n_i} + \frac{1}{2}e_{i,n_i}^2 + \frac{1}{2}\bar{\varepsilon}_{i,n_i}^2 \tag{46}$$

此时,构建 Lyapunov 函数:

$$V_{i,n_i}=V_{i,n_i-1}+\frac{1}{2}e_{i,n_i}^2+\frac{1}{2\boldsymbol{\Gamma}_{i,n_i}}\tilde{\eta}_{i,n_i}^2 \tag{47}$$

根据引理1,可得:

$$\begin{aligned}\dot{V}_{i,n_i} \leqslant & -\frac{3}{4}(A_{ij}+b_i)e_{i,1}^2+(A_{ij}+b_i)g_{i,1}^2e_{i,2}^2- \\ & \frac{3}{4}\sum_{l=2}^{n_i}e_{i,l}^2+\sum_{l=2}^{n_i}g_{i,l}^2e_{i,l+1}^2+\frac{1}{2}\sum_{l=1}^{n_i}\bar{\varepsilon}_{i,l}^2+ \\ & \sum_{l=1}^{n_i}\dot{\zeta}_{i,l}[g_{i,l}N(\zeta_{i,l})\dot{\zeta}_{i,l}+1]+\sum_{l=1}^{n_i}\varepsilon_{i,l}\eta_{i,1}\gamma_{i,1}+ \\ & \sum_{l=1}^{n_i}\left[\frac{\sigma_{i,2}\|\mathrm{e}\|}{2\boldsymbol{\Gamma}_{i,1}}(\eta_{i,2}^2-\tilde{\eta}_{i,2}^2)\right]+ \\ & e_{i,n_i}\Big[g_{i,n_i}\Big(\sum_{\Omega_1}u_0+\sum_{\Omega_2}\rho_i\varphi_iu_0+\sum_{\Omega_3}\bar{u}+\sum_{\Omega_4}\tilde{u}\Big)- \\ & g_{i,n_i}\beta_{i,n_i}\tanh(\lambda_{i,n_i}e_{i,n_i})\Big]+ \\ & e_{i,n_i}\eta_{i,n_i}\|\boldsymbol{\Theta}_{i,n_i}\|\tanh\left(\frac{e_{i,n_i}\|\boldsymbol{\Theta}_{i,n_i}\|}{\gamma_{i,n_i}}\right)+ \\ & \varepsilon_{i,n_i}\eta_{i,n_i}\gamma_{i,n_i}+\frac{1}{2}\sum_{l=1}^{n_i-1}d_{i,l}^2\end{aligned} \tag{48}$$

由式(48)可得控制律和更新律:

$$\begin{aligned}u_0= & N(\zeta_{i,n_i})\Big[\hat{\eta}_{i,1}\|\boldsymbol{\Theta}_{i,n_i}\|\tanh\left(\frac{e_{i,n_i}\|\boldsymbol{\Theta}_{i,n_i}\|}{\gamma_{i,n_i}}\right)+ \\ & e_{i,n_i}+\sum_{\Omega_3}\bar{u}+\sum_{\Omega_4}\tilde{u}-\beta_{i,n_i}\tanh(\lambda_{i,n_i}e_{i,n_i})\Big]\end{aligned} \tag{49}$$

$$\begin{aligned}\dot{\zeta}_{i,n_i}= & e_{i,n_i}^2+e_{i,n_i}\sum_{\Omega_4}\tilde{u}+e_{i,n_i}\hat{\eta}_{i,1}\|\boldsymbol{\Theta}_{i,n_i}\|\tanh\left(\frac{e_{i,n_i}\|\boldsymbol{\Theta}_{i,n_i}\|}{\gamma_{i,n_i}}\right)- \\ & e_{i,n_i}\beta_{i,n_i}\tanh(\lambda_{i,n_i}e_{i,n_i})+e_{i,n_i}\sum_{\Omega_3}\bar{u}\end{aligned} \tag{50}$$

$$\dot{\hat{\eta}}_{i,n_i}\leqslant\boldsymbol{\Gamma}_{i,n_i}\left[e_{i,n_i}\|\boldsymbol{\Theta}_{i,n_i}\|\tanh\left(\frac{e_{i,n_i}\|\boldsymbol{\Theta}_{i,n_i}\|}{\gamma_{i,n_i}}\right)-\sigma_{i,n_i}\|e\|\hat{\eta}_{i,n_i}\right] \tag{51}$$

将式(49)~(51)代入式(48),可得:

$$\begin{aligned}\dot{V}_{i,n_i}\leqslant & -(A_{ij}+b_i)e_{i,1}^2-\frac{3}{4}\sum_{l=2}^{n_i}e_{i,l}^2+(A_{ij}+b_i)g_{i,1}^2e_{i,2}^2+ \\ & \sum_{l=2}^{n_i}g_{i,l}^2e_{i,l+1}^2+\frac{1}{2}\sum_{l=1}^{n_i}\bar{\varepsilon}_{i,l}^2+\sum_{l=1}^{n_i}\varepsilon_{i,l}\eta_{i,1}\gamma_{i,1}- \\ & e_{i,n_i}d_{i,n_i-1}+\sum_{l=1}^{n_i}\dot{\zeta}_{i,l}[g_{i,l}N(\zeta_{i,l})\dot{\zeta}_{i,l}+1]+ \\ & \sum_{l=1}^{n_i}\left[\frac{\sigma_{i,2}\|e\|}{2\boldsymbol{\Gamma}_{i,1}}(\eta_{i,2}^2-\tilde{\eta}_{i,2}^2)\right]\end{aligned} \tag{52}$$

此时,令 $c_{i,n_i}=\min\left\{\frac{3}{2},\varepsilon,\sigma_{i,n_i}\right\}$和 $\Xi_{n_i}=\frac{\sigma_{i,n_i}\|e\|}{2\boldsymbol{\Gamma}_{i,n_i}}\eta_{i,n_i}^2+\varepsilon\eta_{i,n_i}\gamma_{i,n_i}+\frac{1}{2}\bar{\varepsilon}_{i,n_i}^2+\frac{1}{2}d_{i,n_i-1}^2$,则有:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}(\dot{V}_{i,n_i}\mathrm{e}^{c_{i,n_i}t})\leqslant & -c_{i,n_i}V_{i,n_i}+\Xi_{n_i}+g_{i,n_i}N(\zeta_{i,n_i})\dot{\zeta}_{i,n_i}+ \\ & \dot{\zeta}_{i,n_i}+(A_{ij}+b_i)g_{i,n_i}^2e_{i,n_i+1}^2\end{aligned} \tag{53}$$

式(53)两边同乘以 $\mathrm{e}^{c_{i,n_i}t}$,可得:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}(\dot{V}_{i,n_i}\mathrm{e}^{c_{i,n_i}t})\leqslant & \Xi_{n_i}\mathrm{e}^{c_{i,n_i}t}+g_{i,n_i}N(\zeta_{i,n_i})\dot{\zeta}_{i,n_i}\mathrm{e}^{c_{i,n_i}t}+ \\ & \dot{\zeta}_{i,n_i}\mathrm{e}^{c_{i,n_i}t}+(A_{ij}+b_i)g_{i,n_i}^2e_{i,n_i+1}^2\mathrm{e}^{c_{i,n_i}t}\end{aligned} \tag{54}$$

对式(54)两边同时积分,可得:

$$\begin{aligned}V_{i,n_i}(t)\leqslant & \frac{\Xi_{n_i}}{c_{i,n_i}}+\mathrm{e}^{-c_{i,n_i}t}\int_0^t[g_{i,n_i}N(\zeta_{i,n_i})+1]\dot{\zeta}_{i,n_i}\mathrm{e}^{c_{i,n_i}t}\mathrm{d}t+ \\ & \mathrm{e}^{-c_{i,n_i}t}(A_{ij}+b_i)\int_0^t[g_{i,n_i}N(\zeta_{i,n_i})+1]\dot{\zeta}_{i,n_i}\mathrm{e}^{c_{i,n_i}t}\mathrm{d}t\end{aligned} \tag{55}$$

4 稳定性分析

为了验证所设计的协同容错控制器的主要结果,现给出以下定理。

定理 考虑一类典型的含有混合执行器故障和多未知控制方向的非线性多智能编队系统式(8),同时满足假设1~3。与此同时,利用虚拟控制律式(17)和式(33),实际控制律式(49)和参数更新律式(18)~(19)、式(34)~(35)以及式(50)~(51),使得闭环系统状态量保持有界,而且保证多智能在执行器故障和未知控制方向共存的约束情形下跟踪参考轨迹 y_r,实现队形的保持,即$\lim\limits_{t\to\infty}y_i-y_r=0$。

证明:构建多智能体编队系统的 Lyapunov 函数,如式(54)所示。

$$V_N=\sum_{i=1}^{n}V_{i,n_i}=\frac{1}{2}\sum_{i=1}^{n}\sum_{q=1}^{m}\left(c_{i,q}e_{i,n_i}^2+\frac{1}{\boldsymbol{\Gamma}_{i,n_i}}\tilde{\eta}_{i,n_i}^2\right) \tag{56}$$

对式(56)求导,整理化简可得:

$$\begin{aligned}\dot{V}_{i,n_i}\leqslant & -\frac{3}{4}(A_{ij}+b_i)e_{i,1}^2+(A_{ij}+b_i)g_{i,1}^2e_{i,2}^2- \\ & \frac{3}{4}\sum_{l=2}^{n_i}e_{i,l}^2+\sum_{l=2}^{n_i}g_{i,l}^2e_{i,l+1}^2+\frac{1}{2}\sum_{l=1}^{n_i}\bar{\varepsilon}_{i,l}^2+ \\ & \sum_{l=1}^{n_i}\dot{\zeta}_{i,l}[g_{i,l}N(\zeta_{i,l})\dot{\zeta}_{i,l}+1]+\sum_{l=1}^{n_i}\varepsilon_{i,l}\eta_{i,1}\gamma_{i,1}+ \\ & \sum_{l=1}^{n_i}\left[\frac{\sigma_{i,2}\|e\|}{2\boldsymbol{\Gamma}_{i,1}}(\eta_{i,2}^2-\tilde{\eta}_{i,2}^2)\right]+ \\ & e_{i,n_i}\Big[g_{i,n_i}\Big(\sum_{\Omega_1}u_0+\sum_{\Omega_2}\rho_i\phi_iu_0+\sum_{\Omega_3}\bar{u}+\sum_{\Omega_4}\tilde{u}\Big)- \\ & g_{i,n_i}\beta_{i,n_i}\tanh(\lambda_{i,n_i}e_{i,n_i})\Big]+ \\ & e_{i,n_i}\eta_{i,n_i}\|\boldsymbol{\Theta}_{i,n_i}\|\tanh\left(\frac{e_{i,n_i}\|\boldsymbol{\Theta}_{i,n_i}\|}{\gamma_{i,n_i}}\right)+\end{aligned}$$

$$\varepsilon_{i,n_i}\eta_{i,n_i}\gamma_{i,n_i} + \frac{1}{2}\sum_{l=1}^{n_i-1} d_{i,l}^2 \tag{57}$$

式(57)可进一步化简为:

$$\dot{V}_{i,n_i} \leqslant -(A_{ij}+b_i)e_{i,1}^2 - \frac{3}{4}\sum_{l=2}^{n_i} e_{i,l}^2 + (A_{ij}+b_i)g_{i,1}^2 e_{i,2}^2 + \sum_{l=2}^{n_i} g_{i,l}^2 e_{i,l+1}^2 + \frac{1}{2}\sum_{l=1}^{n_i}\tilde{\varepsilon}_{i,l}^2 + \sum_{l=1}^{n_i}\dot{\zeta}_{i,l}[g_{i,l}N(\zeta_{i,l})\dot{\zeta}_{i,l}+1] + \sum_{l=1}^{n_i}\left[\frac{\sigma_{i,2}\|e\|}{2\Gamma_{i,1}}(\eta_{i,2}^2-\tilde{\eta}_{i,2}^2)\right] + \sum_{l=1}^{n_i}\varepsilon_{i,l}\eta_{i,1}\gamma_{i,1} - e_{i,n_i}d_{i,n_i-1} \tag{58}$$

令 $c_{i,n_i} = \min\left\{\frac{3}{2},\varepsilon,\sigma_{i,n_i}\right\}$ 和 $\Xi_{n_i} = \frac{\sigma_{i,n_i}\|e\|}{2\Gamma_{i,n_i}}\eta_{i,n_i}^2 + \varepsilon\eta_{i,n_i}\gamma_{i,n_i} + \frac{1}{2}\tilde{\varepsilon}_{i,n_i}^2 + \frac{1}{2}d_{i,n_i-1}^2$,则有:

$$V_{i,n_i}(t) \leqslant \frac{\Xi_{n_i}}{c_{i,n_i}} + e^{-c_{i,n_i}t}\int_0^t [g_{i,n_i}N(\zeta_{i,n_i})+1]\dot{\zeta}_{i,n_i}e^{c_{i,n_i}t}dt + e^{-c_{i,n_i}t}(A_{ij}+b_i)\int_0^t [g_{i,n_i}N(\zeta_{i,n_i})+1]\dot{\zeta}_{i,n_i}e^{c_{i,n_i}t}dt \tag{59}$$

式中,$\Xi_{n_i}/c_{i,n_i}$ 为正常数。

根据式(18)~(19)、式(34)~(35)以及式(50)~(51),结合式(57)可知,所有信号参数有界。虚拟控制律和实际控制律均为有界的。同时,联立式(58)和式(59),可知,跟踪误差 e_{i,n_i} 也为零,即多智能体的轨迹与参考轨迹重合。因此,不仅证明了多智能体系统状态均有界,而且证明了在时间 t 多智能体的运动轨迹与参考轨迹重合。 □

5　数值仿真

为了验证在执行器故障和未知控制方向复合约束下的容错控制方法,利用四组智能体编队子系统进行有限时间容错控制仿真实验,其中每个编队子系统含有 4 个智能体。为了解决复合约束情形下模型不确定性和反步法中的计算复杂性问题,采用鲁棒自适应模糊技术和一阶滑模积分器进行处理。

本节采用典型的含有执行器故障类型中的偏执故障和未知控制方向的二阶非线性系统及实际工程应用中“领航者 - 跟随者”无人机编队系统进行实验。

实例 1　针对实验过程中的多智能体系统,考虑二阶非线性系统模型,如式(60)所示。

$$\begin{cases}\dot{x}_1 = x_2 + 2x_1^2\sin(x_2) + x_1x_2^2 \\ \dot{x}_2 = 2g_{2_1}u_1^F + 2g_{2_2}u_2^F + x_1\cos(x_2) + x_1^2x_2e^{x_2} \\ y = x_1\end{cases} \tag{60}$$

其中,x_1 和 x_2 表示系统的状态,u_1 和 u_2 表示系统的输入,y 表示系统的输出,g_{2_1} 和 g_{2_2} 表示未知非线性函数。仿真初值设定为:$\hat{\theta}_1(0)=\hat{\theta}_2(0)=0$,$\zeta_1=\zeta_1=0$,$x_1(0)=1.5$,$x_2(0)=2$,$g_{2_1}=1.5$,$g_{2_2}=2.5$。执行器故障模型如式(61)和(62)所示。

$$u_1 = \begin{cases}-0.5u_1^F(t) & t\in(0,t+kT] \\ -0.6u_1^F(t)(1.6-e^{-t})u_1^F(t) & t\in(t+kT,2kT]\end{cases} \tag{61}$$

$$u_2 = \begin{cases}0.5u_2^F(t)(1.5-e^{-t})u_2^F(t) & t\in(0,t+kT] \\ 0.6u_2^F(t) & t\in(t+kT,2kT]\end{cases} \tag{62}$$

除此之外,本节以单组智能体编队作为被控对象,且单组之间的智能体为同构,具有相同的特性,其有向拓扑结构如图 1 所示。

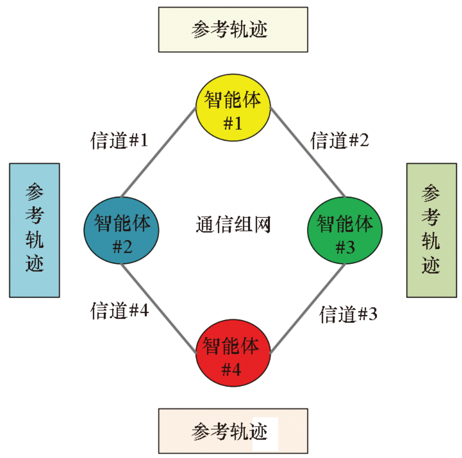

图 1　单组多智能体编队网络拓扑图

Fig. 1　Single-group multi-agent formation network topology diagram

根据上述的初值设定和假设,其仿真结果如图 2 ~ 4 所示。

由图 2 可知,所提出的基于鲁棒自适应模糊技术的协同容错控制方法,使得智能体依旧按照预设的轨迹小误差范围运动,保持良好的稳定特性。同时,通过引入一阶滑模微分器,将追踪误差逐渐趋于零,提高受限情形下系统的容错性能。

由图 3 和图 4 可知,针对非线性多智能体中含有混合执行器故障和多未知控制方向的问题,基于自适应模糊技术,提出的容错控制方法能够使多智能体系统模型参数最终趋于零。

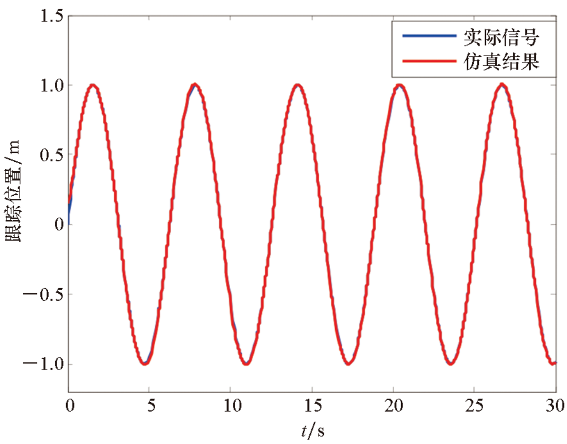

图 2　智能体位置跟踪信号曲线

Fig. 2　Agent position tracking signal curve

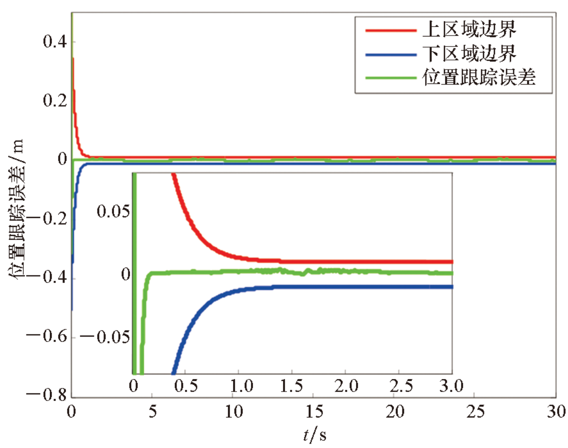

图 3　多智能体系统跟踪误差性能曲线

Fig. 3　Multi-agent system tracking error performance curve

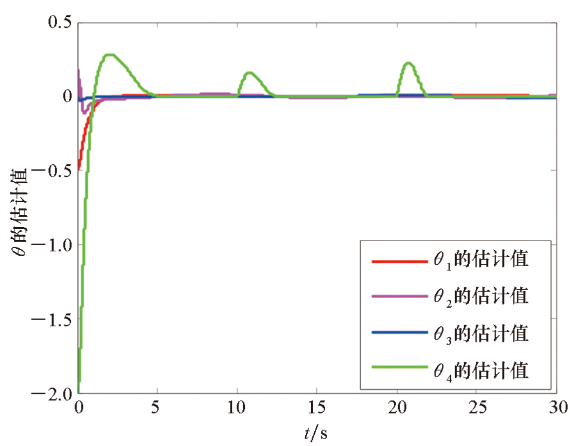

图 4　估计参数 $\theta_i(i=1, 2, 3, 4)$ 的曲线

Fig. 4　Estimated parameter $\theta_i(i=1, 2, 3, 4)$ curve

实例 2　为了进一步说明容错控制方法的有效性和适用性，将 1 个组的编队子系统扩展为 4 个组的编队子系统以作为被控对象，即 4 个组构成的编队系统以同时 4 个组也具有相同的特性，属于同构智能体编队。机器人编队中领航者和跟随者的动力学模型如式(63)和式(64)所示。

$$\begin{cases}\dot{x}_{1,1}(t)=x_{1,2}(t)\\ \dot{x}_{1,2}(t)=2\sin(0.5t)\end{cases}\tag{63}$$

其中，1 表示领航者。

$$\begin{cases}\dot{x}_{i,1}(t)=x_{i,2}(t)\\ \dot{x}_{i,2}(t)=g_{i,n_i}u_i+x_1\sin(x_2)+x_1^2x_2\end{cases}\tag{64}$$

其中，$i=1,2,3,\cdots,16$，变量的含义与系统模型式(8)一致。仿真初值设定为：$M_i=2$ kg，$v_i(0)=0.5$ m/s，$x_{i,1}(0)=0.5$，$x_{i,2}(0)=-0.5$。设置参考轨迹为：$p_{ix}=10t$，$p_{iy}=10\sin(0.5t)$，同时每个智能体的初始位置(p_{i0},p_{i0})和对应的指定位置$(\delta_{ix},\delta_{iy})$的取值见表 2。

表 2　仿真实验参数

Tab. 2　Simulation experiment parameters

单位：m

编号	(p_{i0},p_{i0})	$(\delta_{ix},\delta_{iy})$
智能体#1	(−6, −14)	(0, 0)
智能体#2	(−18, −21)	(−14, 0)
智能体#3	(−18, −29)	(1, 16)
智能体#4	(−25, −35)	(−2, 20)
智能体#5	(−5, −54)	(0, 5)
智能体#6	(−18, −64)	(−6, 2)
智能体#7	(−18, −74)	(1, 24)
智能体#8	(−25, −84)	(−3, 15)
智能体#9	(−5, −64)	(3, 5)
智能体#10	(−15, −74)	(2, 6)
智能体#11	(−15, −84)	(7, 18)
智能体#12	(−20, −94)	(5, 23)
智能体#13	(−4, −74)	(−3, 13)
智能体#14	(−14, −84)	(12, 1)
智能体#15	(−15, −94)	(−7, 25)
智能体#16	(−22, −104)	(−4, 20)

其拓扑结构如图 5 所示。根据初值设定和假设，其仿真实验结果如图 6 ~ 8 所示。

由图 6 可知，控制方法能够提高在执行器故障和未知控制方向共存情形下的容错性能。图 7 针对具有多未知控制方向的多智能体系统，采用分段 Nussbaum 函数进行处理，结合鲁棒自适应模糊技术减小追踪误差，提高控制精度，进而改善在故障情形下的容错性能。图 8、图 9 解决了在混合执行器故障和未知控制方向复合控制情形下的容错控制问题，其位置跟踪误差变化小，而且补偿效果良好。

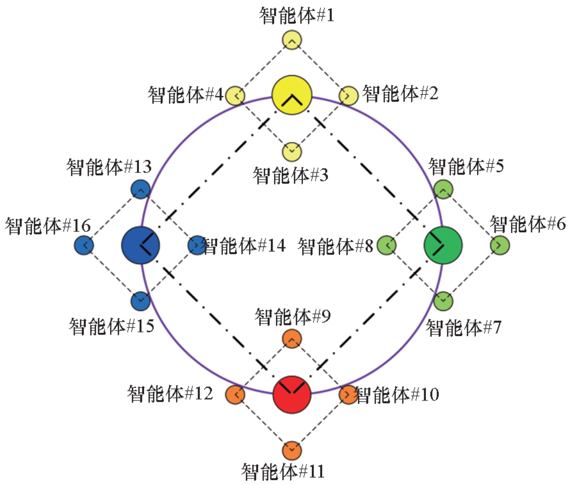

图 5　4 组多智能体编队系统网络拓扑结构图

Fig. 5　Network topology structure diagram of 4 groups of multi-agent formation system

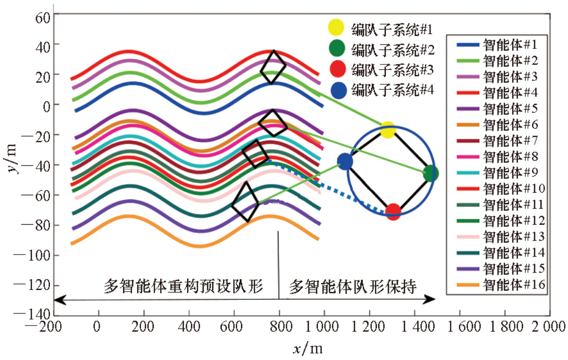

图 6　4 个编队子系统的运动轨迹曲线

Fig. 6　Trajectory curves of 4 formation subsystems

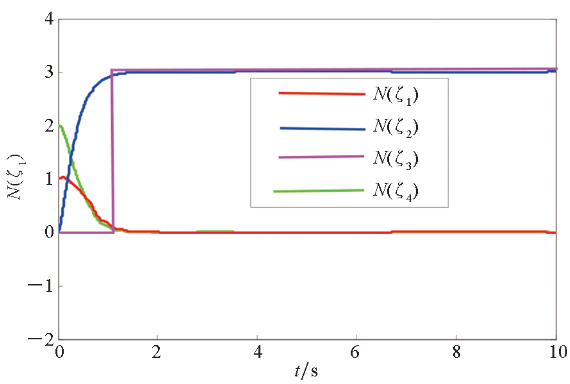

图 7　4 种情形下的 Nussbaum 函数曲线图

Fig. 7　Nussbaum function curves in 4 situations

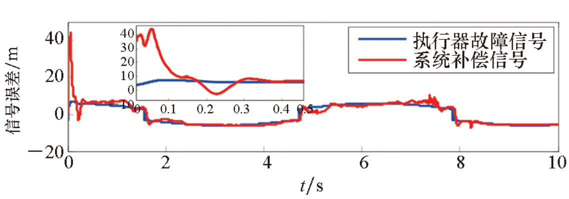

图 8　多智能体系统在执行器故障下的容错性能曲线

Fig. 8　Fault-tolerant performance curve of multi-agent system under actuator fault

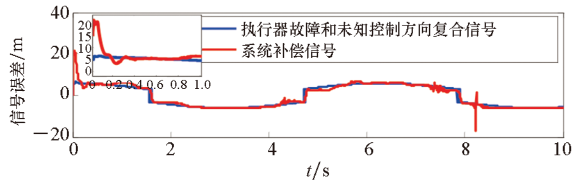

图 9　多智能体系统在复合约束下的容错性能曲线

Fig. 9　Fault-tolerant performance curve of multi-agent system under compound constraints

6　结论

本文针对一类典型的非线性多智能体系统中存在混合执行器故障和多未知控制方向的复合约束问题提出了一种基于鲁棒自适应模糊技术的新颖的协同容错控制方法。该方法将分段 Nussbaum 函数和鲁棒自适应技术结合起来，分别对多未知控制方向和混合执行器故障进行处理，同时利用一阶滑模积分器简化容错控制律的设计过程，将追踪误差限定在预设的允差范围之类，提高了在不同编队结构系统中追踪误差的控制精度和收敛速度。此外，将多智能体规模从由 4 个智能体组成的单组编队系统扩展为由 16 个智能体组成的 4 组编队系统，验证了本文算法的有效性和合理性。主要的贡献如下：

1）提出基于鲁棒自适应模糊控制的协同容错方案，以补偿多种类型的执行器故障损失，而无须任何故障检测和隔离机制。本文控制方法可以补偿多种类型的执行器故障，并且使系统成功稳定下来。此外，即使在执行器出现故障的情况下，该方法也可以确保输出的有界性。因此，可以实现有效、低成本和可靠的系统控制设计的目标。

2）首次研究了针对不确定多智能体系统具有混合执行器故障和部分控制方向未知相结合的问题；基于分段 Nussbaum 函数方法解决了，部分控制增益未知、部分控制增益已知的问题。本研究更具普遍性以及更高的实际应用价值。

3）区别于传统反步技术以及传统动态面反步方法，将一阶滑模微分器技术与反步技术相结合，该方法不仅能够有效地规避冗余的虚拟控制律求导中的“计算爆炸”问题，并且忽略有限时间收敛特性同时满足分离原理，具有更好的控制性能，有利于在实际系统中的应用。

4）在系统设计虚拟控制律过程中，加入鲁棒有界估计方法，利用双曲正切函数的有界性，为参数估计值设定估计界，能够有效地减小系统输出和参考轨迹之间的误差，不仅增强了系统的鲁棒性，而且提高了系统的追踪精度。该方法具有适

用范围广、跟踪精度高、容错性能强等优点。

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