rors in $x$ or $y$ axis and the $R$ direction will never diverge in three typical target paths. It is proved by the results that location and tracking technique besed on distance difference and azimuth measurements is valuable when the target's height need not accurate location.

## 4 Conclusion

The way to track a 3-D moving target by using a receiver without elevation angle measurements has put forward the development of passive location and tracking technique and has widened the range of SOPLAT. From simulation results we see that the location effect is good in paths where the emitter's azimuth and distance are changeful, which shows that height message can be extracted from variable distance and azimuth measurements. Therefore some equipments in the receiver may be simplified and $R$ station will become more flexible. In a word the way proposed in this paper is useful in ECM environment.

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## 利用二坐标 R 站实现运动辐射源的三维跟踪


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**摘　要**　本文提出了一种基于目标斜距两次测量值之差和方位角信息的定位跟踪算法，详细分析了该技术的定位原理和可行性。在辐射源匀速直线运动的条件下，如果能够不测俯仰角而只用方位角和距离差实现对目标的定位与跟踪，将不仅可以简化接收站的设备，而且还能够增强其独立性能，因此本文讨论的算法对于单站被动定位跟踪系统的实用性具有重要意义。本文通过典型目标航迹的计算机仿真，对算法的性能做了评估。



**关键词**　跟踪，方位角，滤波

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# The 3-D Tracking of Moving Emitters Using Two-Coordinate R-Station*

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**Abstract** The paper presents a 3-D location and tracking algorithm based on the emitter's distance difference between two measurements and azimuth message. The location principle and feasibility of the method are expounded in detail. On condition that an emitter moves linearly at a constant speed, if we can locate and track the emitter only by using azimuth and distance difference without any angle of elevation, the equipment in the receiver will be simplified and the independence of the receiver will be Improved as well. Then the algorithm proposed in the paper is of great importance to the practicability of the single observer passive location and tracking system. Performance of this algorithm is evaluated with the help of computer simulation of three typical target paths.

**Key words** Tracking, azimuth, filtering

## 0 Introduction

Today it becomes more and more dangerous to locate and track moving targets by means of active radars due to the development of ECM[1]. Therefore radar experts turn their attention to passive location and tracking technique. One of the important uses of SOPLAT is to locate and track moving emitters with passive method. Generally a target's position is determined by measuring its angle of azimuth and elevation relative to the receiver and the difference of time of arrival between two radio waves that are transmitted by the emitter. However some receivers have not any equipment to measure target's angle of elevation, so it is necessary to do research on the ability of single station to locate and track a moving emitter without any elevation message. This paper will discuss a tracking method only using distance difference and azimuth measurements based on the WMEKF algorithm.

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# 1 Location and Tracking Principle

### 1.1 Special Conditions

a. The target will fly at a constant speed. In other words $v_x$ $v_y$ and $v_z$ will not change.

b. All measuring noises obey Gaussian distribution and are independent of each other.

c. The repetition interval of emitter signal is known.

### 1.2 Location and Tracking Principle

For an emitter in the air, let $(\triangle TOA)^m$ represent the difference of time of arrival (mark $m$ represents measurement). If the emitter moves at a constant speed and the noises are considered, then:

$$(\triangle TOA)^m_{k,k-1} = T + (r_{r,k} - r_{r,k-1})/c + \eta_{\triangle k,k-1} \qquad (1)$$

$$\beta^m_k = \mathrm{tg}^{-1}\frac{y_k - y_r}{x_k - x_r} + \eta_{\beta,k}$$

where: $\eta_{\triangle k,k-1}$, $\eta_{\beta,k}$ are measuring noises of time difference and azimuth.

$T$ is sampling period and $c$ is the speed of light.

$r_r$ is the distance between receiver and target.

The target's coordinate at point $k$ is $(x_k, y_k, z_k)$. R-station's coordinate is $(x_r, y_r, z_r)$.

In the same way distance difference of target to receiver between point $i$ and $i-j(i>j)$ can be easily written as:

$$\begin{aligned}\triangle r_{i,i-j} &= c[(\triangle TOA)^m_{i,i-j} - jT] \\ &= (r_{r,i} - r_{r,i-j}) + \sum_{m=0}^{j-1}\eta_{\triangle r,i-m,i-m-1}\end{aligned}$$

$\eta_{\triangle r} = c\eta_{\triangle}$ is the noise of distance difference that is caused by measuring time difference.

In (1) $(\triangle TOA)^m_{k,k-1}$ contains height message of the target, because:

$$\begin{aligned}r_{r,k} - r_{r,k-1} = {}& \sqrt{(x_k - x_r)^2 + (y_k - y_r)^2 + (z_k - z_r)^2} \\ & - \sqrt{(x_k - Tv_x - x_r)^2 + (y_k - Tv_y - y_r)^2 + (z_k - Tv_z - z_r)^2}\end{aligned}$$

On the one hand the height message contained in $(\triangle TOA)^m$ may be extracted in order to loecte and track a moving emitter without any elevation message, on the other hand we must notice that there is no height message in $\beta^m$, so time difference should be changeful. In other words if the distance has little change in an emitter path, location will certainly fail. According to what we have discussed it can be said that it is possible to track a 3-D moving emitter by only using time difference and azimuth measurement. Computer simulation will prove this.

# 2 Filtering Algorithm

### 2.1 State Function

When an emitter moves at *a* constant speed and the velocity disturbance is $[w_{x,k}, w_{y,k}, w_{z,k}]$, then:

$$X_{k+1} = \Phi X_k + W_k, \text{where: } X_k = [x_k, y_k, z_k, v_{x,k}, v_{y,k}, v_{z,k}]^T$$

$$W_k = [T^2 w_{x,k}/2, T^2 w_{y,k}/2, T^2 w_{z,k}/2, T w_{x,k}, T w_{y,k}, T w_{z,k}]^T$$

$$\Phi = \begin{bmatrix} 1 & 0 & 0 & T & 0 & 0 \\ 0 & 1 & 0 & 0 & T & 0 \\ 0 & 0 & 1 & 0 & 0 & T \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \mathrm{E}[W_k] = 0, \ \mathrm{COV}\{W_k, W_j\} = Q_k \delta_{kj},$$

$$Q_k = \mathrm{E}[W_k W_k^T] = \begin{bmatrix} T^4 q_x^2/4 & 0 & 0 & T^3 q_x^2/2 & 0 & 0 \\ 0 & T^4 q_y^2/4 & 0 & 0 & T^3 q_y^2/2 & 0 \\ 0 & 0 & T^4 q_z^2/4 & 0 & 0 & T^3 q_z^2/2 \\ T^3 q_x^2/2 & 0 & 0 & T^2 q_x^2 & 0 & 0 \\ 0 & T^3 q_y^2/2 & 0 & 0 & T^2 q_y^2 & 0 \\ 0 & 0 & T^3 q_z^2/2 & 0 & 0 & T^2 q_z^2 \end{bmatrix}$$

## 2.2 Measurement Functions

In order to increase the stability, observability and accuracy of the filter we intend to use multiple measurement functions to locate the target. Here we will take four measurement functions for example to study the algorithm. But in fact eight ones are used in simulation in order to get better results.

Number $K$ filtering process will use measuring data at point $K$, $K-N_k$, $K-N_k-I_k$, $K-N_k-I_k-J_k$, $N_k$, $I_k$. $J_k$ are not less than 1. Measurement function:

$$Z_t^m = (X_k) + V_k, \text{ where: } Z_k^m = [\beta_k^m, \beta_{k-N1}^m, \beta_{k-N2}^m, \beta_{k-N3}^m,$$

$$\triangle r_{k,k-N1}, \triangle r_{k-N1,k-N2}, \triangle r_{k-N2,k-N3}]^T$$

In $Z_k^m$ formula: $N_1 = N_k$, $N_2 = N_k + I_k$, $N_3 = N_k + I_k + J_k$,

$$\mathrm{E}[V_k] = 0, \ \mathrm{COV}\{V_k, V_j\} = R_k \delta_{kj},$$

$$R_k = \mathrm{diag}[\sigma_\beta^2, \sigma_\beta^2, \sigma_\beta^2, \sigma_\beta^2, N_k \sigma_{\triangle r}^2, I_k \sigma_{\triangle r}^2, J_k \sigma_{\triangle r}^2]^{\cdot}$$

## 2.3 Filtering Algorithm of Azimuth Measurements

According to reference [2] azimuth measurement function meet with condition of WMEKF algorithm. Let us first obtain $g_{\beta,k}^{N'}(Z_k^m, \hat{X}_{k/k-1})$.

$$\beta_{k-N}^m = \beta_{k-N}^* + \eta_{\beta,k-N},$$

$$\beta_{k-N}^* = \mathrm{tg}^{-1} \frac{y_k - NTv_{y,k} - y_r}{x_k - NTv_{x,k} - x_r} = h_\beta^N(X_k) \tag{2}$$

To make formula (2) pseudo linearization, it can be written as: $H_i(Z_k^*) X_k' = 0$, where: $H_i(Z_k^*)$ is:

$$[\sin\beta_{k-N}^*, \ -\cos\beta_{k-N}^*, \ 0, \ -NT\sin\beta_{k-N}^*, NT\cos\beta_{k-N}^*, 0]$$

$$X_k' = [x_k - x_r, \ y_k - y_r, \ z_k - z_r, \ v_{x,k}, \ v_{y,k}, \ v_{z,k}]^T$$

If $\beta^*_{k-N}=\beta^m_{k-N}-\eta_{\beta,k-N}$ is put in and suppose cos $\eta_{\beta,k-N}\approx 1$, then: $H_i(Z^m_k)X'_k=T_i(Z^m_k,X_k,V_k)$. where $H_i(Z^m_k)$ is $H_i(Z^*_k)$ when $*$ is replaced by $m$. According to reference [2],

$$g_{ki}(Z^m_k,\hat{X}_{k/k-1})=-\frac{Z^m_{ki}-h_i(\hat{X}_{k/k-1})}{H_i(Z^m_k)\hat{X}_{k/k-1}}H_i(Z^m_k),\text{let:}$$

$$Z^m_{ki}=\beta^m_{k-N},\ h_i(\hat{X}_{k/k-1})=\mathrm{tg}^{-1}\frac{\hat{y}_{k/k-1}-y_r-NT\hat{v}_{y,k/k-1}}{\hat{x}_{k/k-1}-x_r-NT\hat{v}_{x,k/k-1}}$$

$$\hat{X}_{k/k-1}=[\hat{x}_{k/k-1}-x_r,\hat{y}_{k/k-1}-y_r,\hat{z}_{k/k-1}-z_r,\ \hat{v}_{x,k/k-1},\hat{v}_{y,k/k-1},\hat{v}_{z,k/k-1}]^T$$

We obtain $g^N_{\beta,k}(Z^m_k,\hat{X}_{k/k-1})$.

Four $g^N_{\beta,k}(Z^m_k,\hat{X}_{k/k-1})$ $(N=0,N_k,N_k+I_k,N_k+I_k+J_k)$ form a $4\times 6$ matrix $g_{\beta,k}(Z^m_k,\hat{X}_{k/k-1})$.

And: $\mu_{ki}=\dfrac{T_i(Z^m_k,X_k,V_k)}{H_i(Z^m_k)\hat{X}_{k/k-1}}$, where $i=1,2,3,4$ are corresponding to $N=0,N_k,N_k+I_k,N_k+I_k+J_k$.

then: $U_k=\mathrm{diag}\left[\dfrac{1}{1-\mu_{k1}},\ \cdots\cdots,\ \dfrac{1}{1-\mu_{k4}}\right]$,

$$g'_{\beta,k}(Z^m_k,\hat{X}_{k/k-1})=U_k g_{\beta,k}(Z^m_k,\hat{X}_{k/k-1}),$$

$$R'_{\beta,k}=U_kR_{\beta,k},U^T_k=\mathrm{diag}\left[\frac{\sigma^2_\beta}{(1-\mu_{k1})^2},\cdots,\frac{\sigma^2_\beta}{(1-\mu_{k4})^2}\right]$$

### 2.4 Filtering Algorithm of Distance Difference Data

The measuring functions of distance difference do not meet with condition of WMEKF algorithm, so we shall have to use EKF algorithm for filtering.

EKF is similar to WMEKF, $g^N_{\Delta r,k}(Z^m_k,\hat{X}_{k/k-1})$ is the local derivative in EKF.

$$h^N_{\Delta r,k}(X_k)=r_{r,k-k2}-r_{r,k-k1},$$

$$\frac{\partial h^N_{\Delta r,k}(X_k)}{\partial x_k}=\frac{x_k-x_r-k_2Tv_{x,k}}{r_{r,k-k2}}-\frac{x_k-x_r-k_1Tv_{x,k}}{r_{r,k-k1}} \tag{3}$$

$$\frac{\partial h^N_{\Delta r,k}(X_k)}{\partial v_{x,k}}=k_1T\frac{x_k-x_r-k_1Tv_{x,k}}{r_{r,k-k1}}-k_2T\frac{x_k-x_r-k_2Tv_{x,k}}{r_{r,k-k2}} \tag{4}$$

To replace $x$ with $y$ or $z$ (including marks) in (3) and (4), we may get $\dfrac{\partial h^N_{\Delta r,k}(X_k)}{\partial y_k}$ or $\dfrac{\partial h^N_{\Delta r,k}(X_k)}{\partial z_k}$ and $\dfrac{\partial h^N_{\Delta r,k}(X_k)}{\partial v_{y,k}}$ or $\dfrac{\partial h^N_{\Delta r,k}(X_k)}{\partial v_{z,k}}$. In which

① $k_1=N_k$, $k_2=0$. ② $k_1=N_k+I_k$, $k_2=N_k$,

③ $k_1=N_k+I_k+J_k$, $k_2=N_k+I_k$.

Three values of $k_1$ and $k_2$ form a $3\times 6$ matrix $g_{\Delta r,k}(Z^m_k,\hat{X}_{k/k-1})$, where each $1\times 6$ submatrix —— $g^N_{\Delta r,k}(Z^m_k,\hat{X}_{k/k-1})$ is written as:

$$\left[\frac{\partial h^N_{\Delta r,k}(X_k)}{\partial x_k},\frac{\partial h^N_{\Delta r,k}(X_k)}{\partial y_k},\frac{\partial h^N_{\Delta r,k}(X_k)}{\partial z_k},\frac{\partial h^N_{\Delta r,k}(X_k)}{\partial v_{x,k}},\ \frac{\partial h^N_{\Delta r,k}(X_k)}{\partial v_{y,k}},\frac{\partial h^N_{\Delta r,k}(X_k)}{\partial v_{z,k}}\right]_{X_m=\hat{X}_{m/n-1}}$$

And $R_{\triangle r,k}=\text{diag}[N_k\sigma^2_{\triangle r},\ I_k\sigma^2_{\triangle r},\ J_k\sigma^2_{\triangle r}]$.

### 2.5 The Application of Filtering Formulae

WMEKF algorithm is introduced as follows:

$$K_k = P_{k/k-1}g_k'^T[g_k'P_{k/k-1}g_k'^T + R_k']^{-1}$$
$$P_{k/k} = [I - K_k g_k']P_{k/k-1}$$
$$\hat{X}_{k/k} = \hat{X}_{k/k-1} + K_k[Z_k^m - h(\hat{X}_{k/k-1})]$$
$$\hat{X}_{k+1/k} = \Phi\hat{X}_{k/k}$$
$$P_{k+1/k} = \Phi P_{k/k}\Phi^T + Q_k$$

where: $g_k'$ is $g_k'(Z_k^m,\hat{X}_{k/k-1})$, Let: $g_k'(Z_k^m,\hat{X}_{k/k-1})=[g'^T_{\beta,k}(Z_k^m,\hat{X}_{k/k-1}),\ g^T_{\triangle r,k}(Z_k^m,\hat{X}_{k/k-1})]^T$

$$R_k' = \begin{bmatrix} R_{\beta,k} & 0 \\ 0 & R_{\triangle r,k} \end{bmatrix}$$

$$h(\hat{X}_{k/k-1}) = [h_\beta^T(\hat{X}_{k/k-1}),\ h^T_{\triangle r}(\hat{X}_{k/k-1})]^T.$$

The original values of the filter are chosen by using following way.

The estimated distance $\hat{R}_o$ between emitter and receiver can be predicted according to the maximum effective distance of receiver. At the same time the target's height $\hat{z}$ can also be estimated. We choose:

$$\hat{x}_o = \sqrt{\hat{R}_o^2 - \hat{z}^2}\cos\beta_o^m + x_r,\quad \hat{y}_o = \sqrt{\hat{R}_o^2 - \hat{z}^2}\sin\beta_o^m + y_r, \hat{z}_o = \hat{z} + z_r,$$
$$(\hat{v}_{xo},\hat{v}_{yo},\hat{v}_{zo}) = 0.5(v_{x\max},v_{y\max},v_{z\max})$$
$$P_{o/o} = \text{diag}[3(\text{km})^2,\ 3(\text{km})^2,\ 3(\text{km})^2,\ (100\text{m/s})^2,\ (100\text{m/s})^2,\ (100\text{m/s})^2].$$

## 3 Computer Simulation Test

### 3.1 Test Conditions and Environment

Receiver station address is (30,0,0) km.

Initial tracking point is (50,60,10) km.

Maximum speed is (−500,−500,0) m/s.

Path ① (−300,−300,0) m/s ②(−450,0,0) m/s ③(0,−450,0)m/s.

$\sigma_\beta=2\text{mrad}$, $\sigma_{\triangle r}=5\text{m}$, $q_x=q_y=20\text{m/s}^2$, $q_z=10\text{m/s}^2$.

$\hat{R}_o=70\text{km}, \hat{z}_o=15\text{km}$, Thirty times of Monte Carlo test.

### 3.2 Analyses of Test Results

a. Location and tracking effects vary from paths. Path ③ reaches best result because of its changeful distance and azimuth. Location effect of path ① is nearly the same as that of path ③ except for larger square deviation. Nevertheless its location errors at $x$ or $y$ axis and $R$ direction are still limited within 1km. Because distance between emitter and receiver has little change in path ② so we fail to locate and track the moving emitter in this path.

b. When a receiver locate and track a 3-D emitter moving at a constant speed only by using distance and azimuth measurements, the maxlmum error is in the height while er

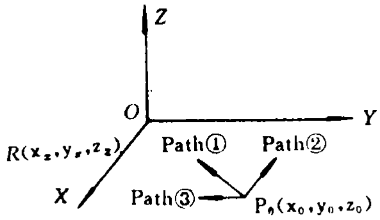


Fig. 1  Target Path

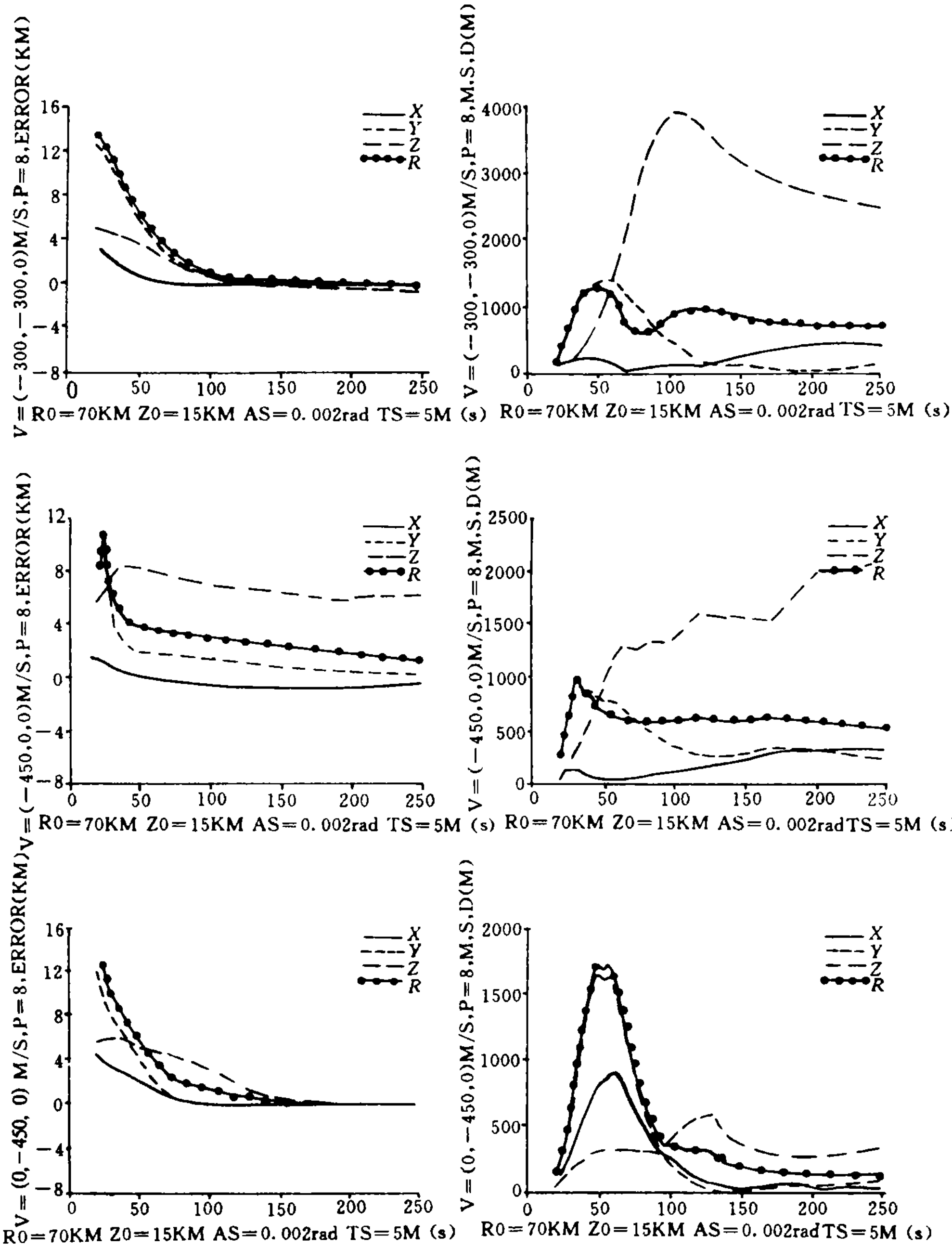


Fig. 2  Tracking Error

rors in $x$ or $y$ axis and the $R$ direction will never diverge in three typical target paths. It is proved by the results that location and tracking technique besed on distance difference and azimuth measurements is valuable when the target's height need not accurate location.

## 4 Conclusion

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**摘　要**　本文提出了一种基于目标斜距两次测量值之差和方位角信息的定位跟踪算法，详细分析了该技术的定位原理和可行性。在辐射源匀速直线运动的条件下，如果能够不测俯仰角而只用方位角和距离差实现对目标的定位与跟踪，将不仅可以简化接收站的设备，而且还能够增强其独立性能，因此本文讨论的算法对于单站被动定位跟踪系统的实用性具有重要意义。本文通过典型目标航迹的计算机仿真，对算法的性能做了评估。

**关键词**　跟踪，方位角，滤波

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